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Cassava Chips Production System in Max-Plus Algebra

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ABSTRACT

The research discussed here relates to the cassava chips production system. The production system was analyzed using petri net and algebra of maxplus. The goal is to obtain information on the completion time for each process and to know the estimated time of completion of production each day. Based on the data obtained, it can be described the flow of the production process and then formed a graph so that it can be formed a petri net using the Woped 3.1.0 tool. From the results of the petri net then analyzed and can be built coverability tree. So that, the completion time for each process and the estimated time for completion of production each day can be known.

1. Introduction

Indonesia is a country known for its many islands, various ethnic groups and cultures. This diversity causes a variety of culinary in Indonesia. Through social media, we can see many things related to Indonesian culinary which are well known to people from various countries around the world. Not only reviews about various kinds of culinary products, but also includes how to cook, process, package and use them in the business world. Culinary products in Indonesia are generally in the form of various snacks, food, or drinks.

Beside various culinary delights, Indonesia is also known to have a variety of fruits, spices, and tubers. Tubers are plants that grow on the ground or in the ground which can also be used as basic ingredients for food or drink. Several types of tubers that thrive in Indonesia are potatoes, carrots, sweet potatoes, cassava, etc.

Cassava, as one of the tubers that is easy to grow in Indonesia, is very popular

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in the Indonesian people with its various refined products (S. N. Aini, Syafi'i, & Kuntadi, 2014). This is because cassava is one of the largest agricultural food product in Indonesia (Q. Aini, Yulianto, & Amalia, 2021). Most people know that cassava is a staple food that makes carbohydrates and that its leaves are vegetables. The cassava plant (*Manihot Utilissima*) can reach 7 meters in height with a number of root branches and then enlarge into edible root tubers. The average tuber size is 2-3 cm in diameter and 50-80 cm, depending on the clone/cultivar. The inside of the tuber is white or yellowish (Sari et al., 2022; Utama & Rukismono, 2018)

One of the typical refined products of cassava that is very popular with consumers is cassava chips. This is because it tastes good, is practical, is suitable to be enjoyed in all situations, and the price is affordable (Harsita & Amam, 2019). So that cassava chips can be used as a promising business potential if they are packaged in various attractive shapes and processed with various flavors. As according to (Soebiyakto & Alfiana, 2017) in their article that the profit of selling packaged cassava chips reaches 45%. Therefore, the flow of the cassava chips production process is very interesting to research.

Thus, this research is related to the cassava chips production system. The production system was analyzed using petri net. Next, analyzed with algebra of max plus. There is related research from (Kansal & Dabas, 2021; Karadogan, 2021; Maure, Ningsi, & Nay, 2021). While max-plus algebra is the algebraic structure of the mathematical system max=$\mathbb{R}\cup\{-\infty\}$ which is equipped with the operation, which is maximum (denoted max) and the operation, which is ordinary addition (denoted +)(Qin & Liu, 2021; Suroto, 2021). There are several previous research results that have used petri nets and max plus algebra in their research, such as research on petri net simulations in the fermented milk production process (Pramesthi, 2021); Petri net design and max-plus algebra on the blessed milk soyghurt production system (Nurmalitasari & Iswahyuni, 2019); The research from (Qin & Liu, 2021); (Pamuk, 2015); and (Konigsberg, 2013). However, there are differences between this study and the previous studies, mainly in the purpose of using the petri net. Also, using algebra of max plus .

This research is to form a graph of the cassava chips production system, petri net and coverability tree and to obtain information on the completion time for each process in the cassava chips production system every day, and to find out Estimated production completion time each day. In addition, this study begins with secondary data collection and uses the library method. Furthermore, based on the data obtained, it can be described the flow of the cassava chips production process and then formed a graph so that the petri net can be formed using the Woped 3.1.0 tool. From the results of the petri net then analyzed and can be built coverability tree. Model of the cassava chips production system was analyzed to obtain information on the completion time for each process in the cassava chips production system everyday and the estimated time of completion of production every day.

2. Method

This research on cassava chips production system with max-plus algebra uses library methods and secondary data collection. The cassava chips production process flow that has been obtained (citation) is then formed using a petri net and analyzed using algebra of max-plus. The following are the stages in this research:

1. Collecting data on the flow of the cassava chips production process,

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2. Draw a graph based on the flow of the cassava chips production process,
3. Establish and analyze a petri net and build a coverability tree for the cassava chips production system,
4. Create a max-plus algebraic model of the cassava chips production system that has been formed, and
5. Max-plus algebraic model analysis for system of cassava chips production

The following is a flow of research stages:

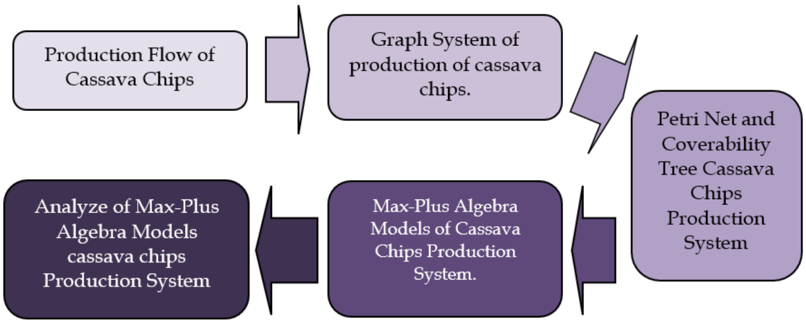

Figure 1. Research Stage Flow of Cassava Chips Production System.

3. Results and Discussion

3.1. Production Flow of Cassava Chips

Assumption:

1. Assumptions in the production flow of cassava chips are that the production system never experiences disturbances, human resources are always ready (stand by), electricity is never interrupted, and the availability of materials is always available.
2. The assumption that in this study the producer of cassava chips is a home industry producer.
3. The assumption of the weight of cassava required in 1 production is 3000 grams, after peeling and washing the weight becomes 2500 grams, after being sliced and dried from the soaking process the weight of cassava becomes 2400 grams and after frying the weight of cassava chips becomes 2100 grams.
4. Assume that the taste of cassava chips is only 1 flavor.
5. Assuming the weight of cassava chips that have been packaged with each package weighing 25 grams/pack or available84 packs per day from 1 time of production.

This following is figure of cassava chips production stage.

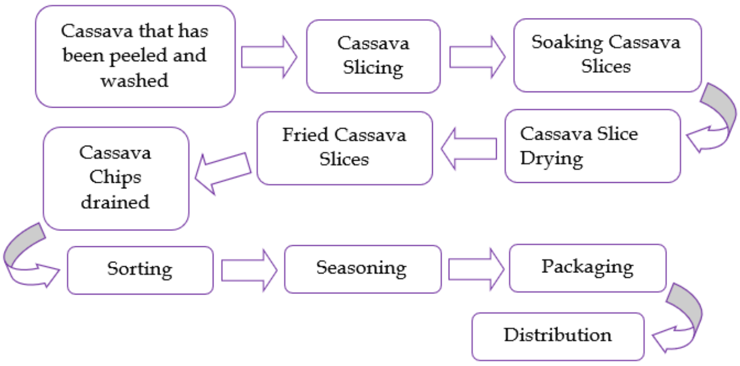

Figure 2. Cassava Chips Production Stage

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3.2. *Graph of Cassava Chips Production System*

Based on the figure of the cassava chips production flow in Figure 2, the graph of the cassava chips production system can be described as follows:

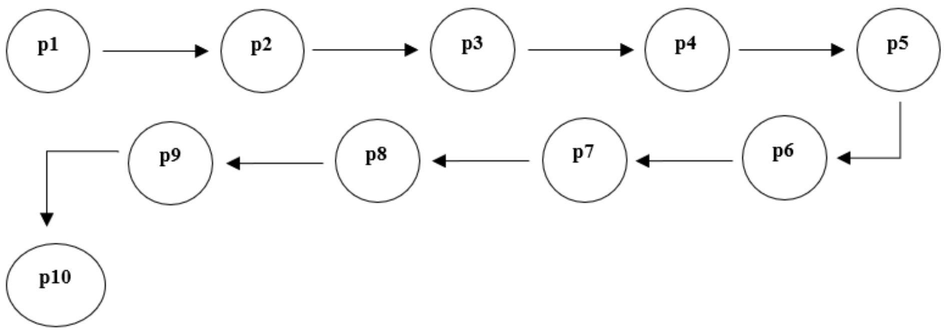

Figure 3. Cassava Chips Production System Graph

The graph for the cassava chips production system can be seen in Figure 3, which has 10 places with the following information:

1. Place of 1 (p1) is a place for peeling and washing cassava,
2. Place of 2 (p2) is a place for slicing cassava,
3. Place of 3 (p3) is a place for soaking cassava slices,
4. Place of 4 (p4) is a place for drying cassava slices
5. Place of 5 (p5) is the place for fried cassava slices
6. Place of 6 (p6) is the place where the cassava chips are drained,
7. Place of 7 (p7) is a place for sorting cassava chips,
8. Place of 8 (p8) is a place for seasoning,
9. Place of 9 (p9) is the place of packaging, and
10. Place 10 (p10) is a distribution place.

3.3. *Petri Net and Coverability Tree Production System of Cassava Chips*

3.3.1. *Petri Net*

Based on Figure 3, it can be formed a petri net using woped 3.1.0. Below can be described petri net cassava chips production system:

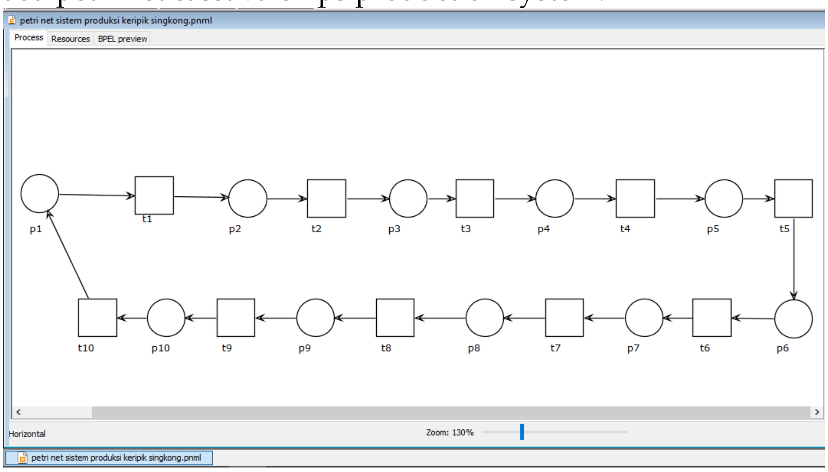

Figure 4. Petri Net Cassava Chips Production System

Petri net cassava chips production system has 10 places and 10 transitions. The following is a description of each place and transition:

Table 1. Description of Petri Net Cassava Chips Production System

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Notation of Each Place	Notation Description of Each Place	Notation of Each Transition	Notation Description of Each Transition
p1	Place for peeling and washing cassava	t1	Time of cassava peeling and washing
p2	Place for slicing cassava	t2	Time of slicing cassava
p3	Place for soaking cassava slices	t3	Time of Soaking cassava slices
p4	Place for drying cassava slices	t4	Time of drying cassava slice
p5	Place fried cassava slices	t5	Time of frying cassava slices
p6	Place drained cassava chips	t6	Time of draining cassava chips
p7	Cassava chips sorting place	t7	Time of sorting cassava chips
p8	Seasoning place	t8	Time of seasoning cassava chips
p9	Packing place	t9	Time of Packing
p10	Place of distribution	t10	Time of Distribution

The petri net is given 1 token in place 1, then the petri net figure is as follows:

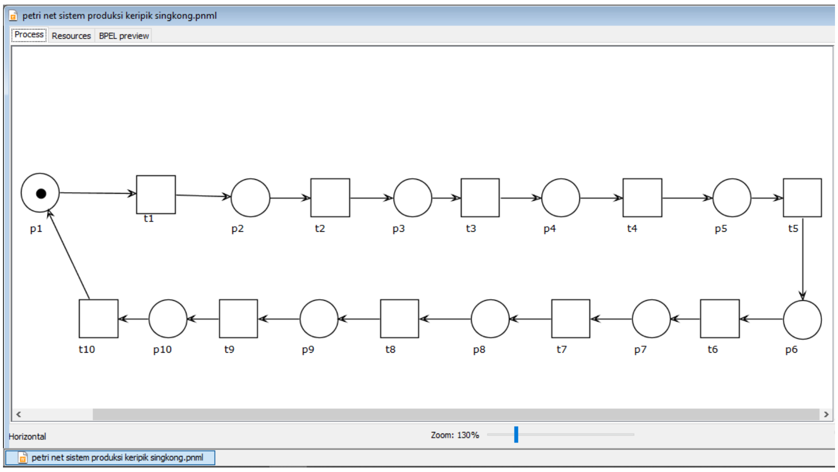

Figure 5. Petri Net Cassava Chips Production System Given 1 Token in Place 1

Furthermore, a petri net analysis of the cassava chip production system was carried out which had been given 1 token in place 1 as follows:

1. This Petri net cassava chip production system has 10 places that can be written as the set p = {p1, p2, p3, p4, p5, p6, p7, p8, p9, p10} and has 10 transitions that can be written as the set t = {t1, t2, t3, t4, t5, t6, t7, t8, t9, t10}.
2. Based on the petri net figure of the cassava chip production system that has been given 1 token in place 1, it can be determined the initial state of the petri net, namely:
$$x_0 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
3. From the initial state that has been given, it can be seen that the enabled transition is transition t1, the following is a petri net figure:

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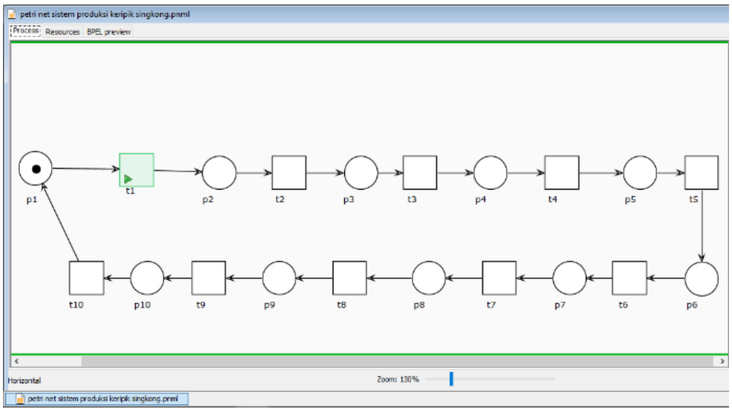

Figure 6. The Enabled Transition Is Transition t1

4. If the transition t1 difire, then the state becomes $x_1 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$.

   The Petri net looks like this:

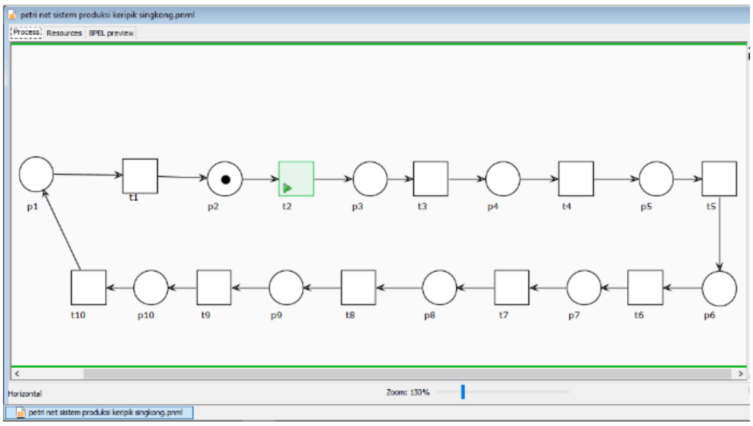

Figure 7. The Transition Enabled Is Transition t2

   The transition that is enabled in the above state is transition t2.

5. If the transition t2 difire, then the state becomes $x_2 = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$.

   The Petri net figure becomes:

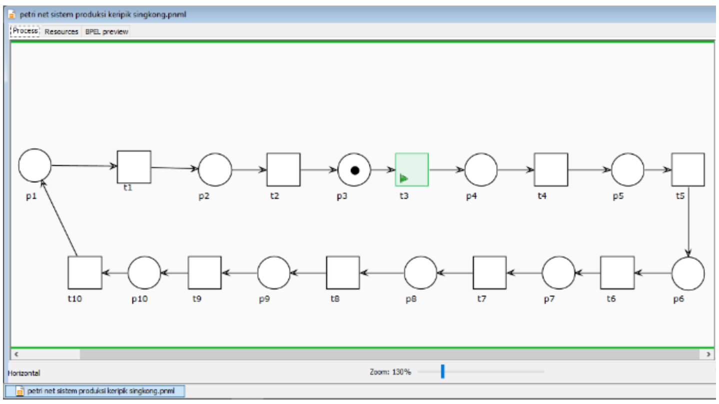

Figure 8. The Transition Enabled Is Transition t3

   The transition that is enabled in the above state is transition t3.

6. If the transition t3 difire, then the state becomes $x_3 = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$.

   The petri net figure is

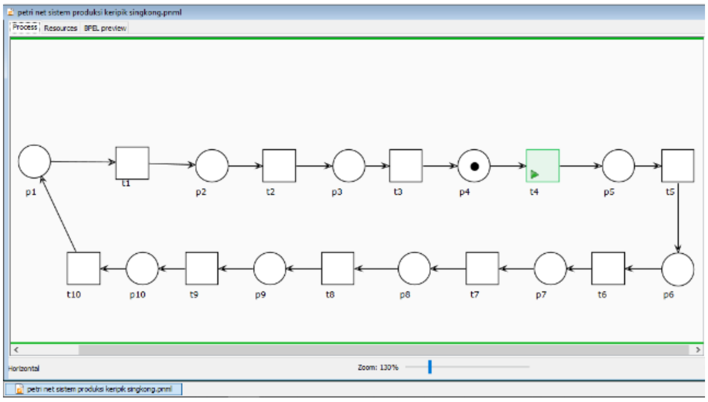

Figure 9. The Transition Enabled Is Transition t4

   The transition that is enabled in the above state is transition t4.

7. If the transition t4 difire, than the state becomes $x_4 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$

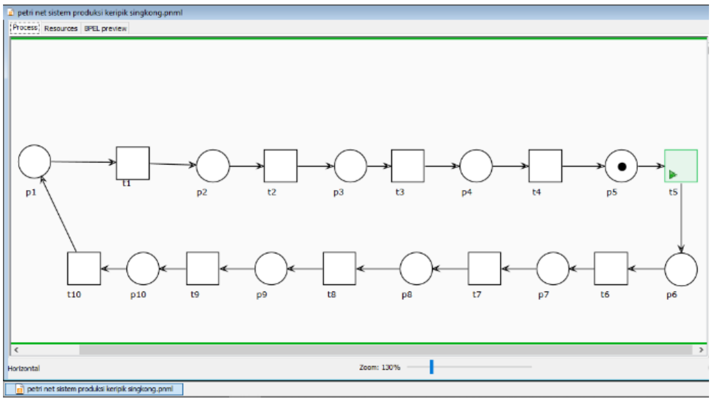

Figure 10. The Transition Enabled Is Transition t5

   The transition that is enabled in the above state is transition t5.

8. If the transition t5 difire, then the state becomes $x_5 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]^T$.

   The petri net figure is

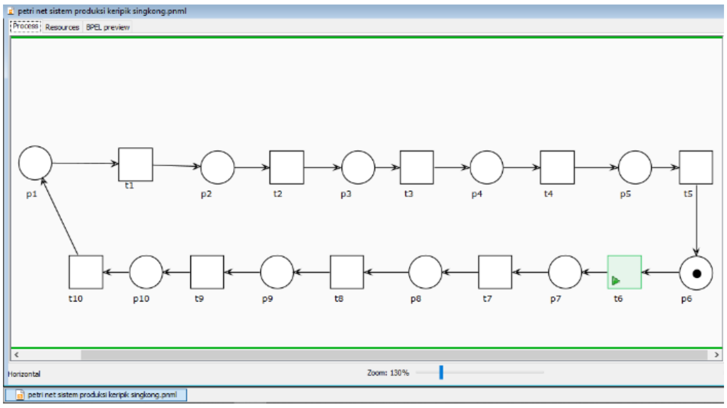

Figure 11. The Transition Enabled Is Transition t6

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Transition t6 is an enabled transition

9. Whereas t6 difire when the transition state number 8, then the situation becomes $x_6 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T$.

   The petri net figure is

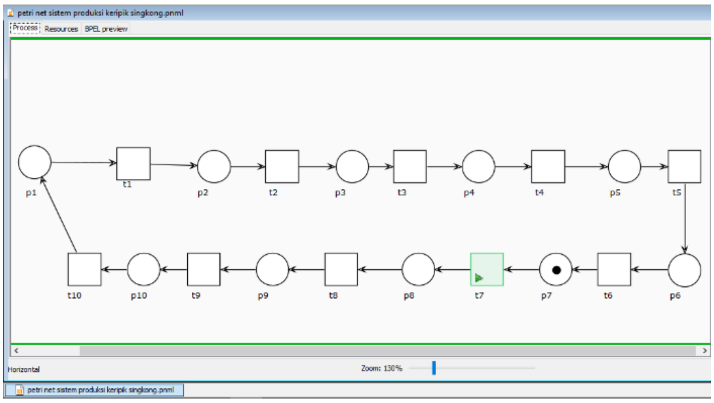

Figure 12. The Transition Enabled Is Transition t7

   The transition that is enabled in the above state is transition t7.

10. If transition t7 difire, then the state becomes $x_7 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]^T$.

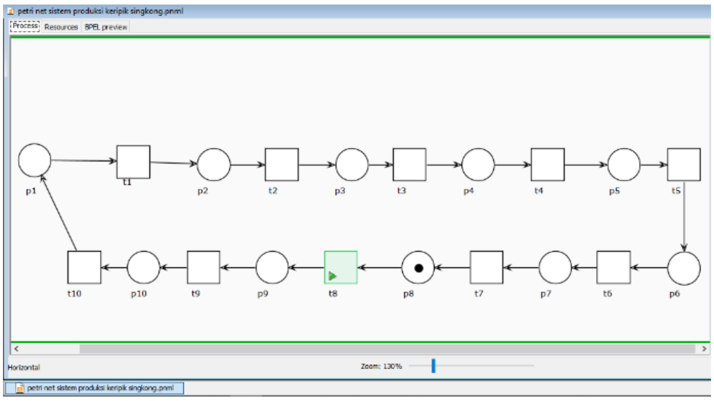

Figure 13. The Transition Enabled Is Transition t8

   Transition t8 is an enabled transition.

11. If transition t8 difire, then the state becomes $x_8 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^T$

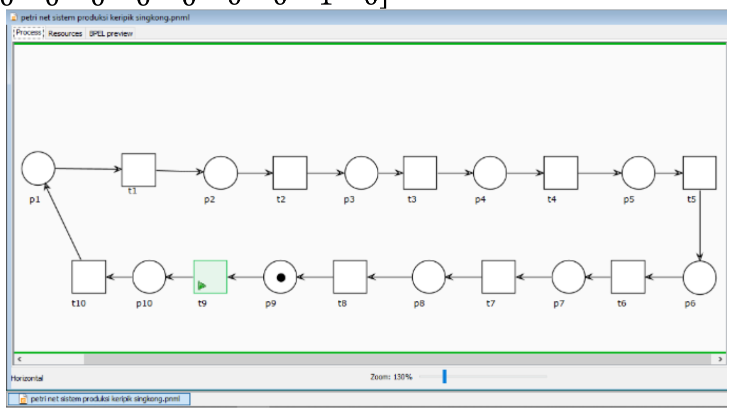

Figure 14. The Transition Enabled Is Transition t9

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Transition t9 is an enabled transition.

12. If transition t9 difire, then the state becomes
$x_9 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]^T$.

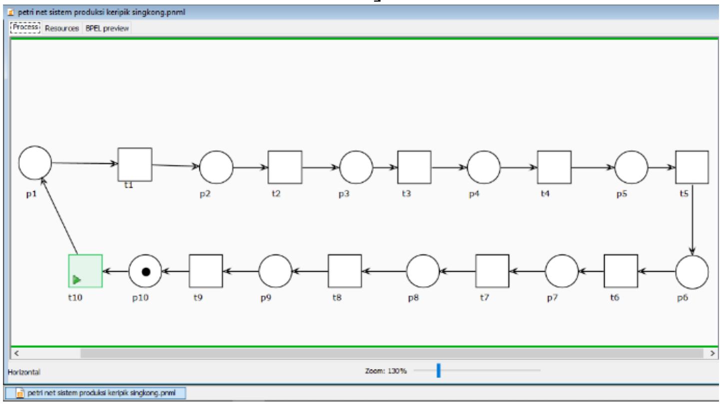

Figure 15. The Transition Enabled Is Transition t10

Transition t10 is an enabled transition.

13. Whereas t10 difire when the transition state number 12, then the situation becomes $x_{10} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$.
Transition t1 is an enabled transition. This state is back to initial state :
$$x_1 = x_{10} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
Thus, the process will repeat continuously.

3.3.2. *Coverability Tree*

Based on the petri net analysis, the coverability tree is as follows:

$$x_0 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_1 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_2 = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_3 = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_4 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_5 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_6 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_7 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]^T$$
$$\downarrow$$
$$x_8 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^T$$
$$\downarrow$$
$$x_9 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]^T$$

3.4. *Analysis Model and Analysis System of Cassava Chips Production*

Based on the results of the petri net, the cassava chips production system shows that it has 10 places that can be written as the set p = {p1, p2, p3, p4, p5, p6, p7, p8, p9, p10} and has 10 transitions that can be written as the set. t = {t1, t2, t3, t4, t5, t6, t7, t8, t9, t10}. To determine the time and length of the cassava chips production process, the results of the petri net were solved using algebra of max-plus. Before

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forming the algebra of max plus model of the cassava chips production system, first write down the definition of each variable used in the petri net of the cassava chips production system as follows:

Table 2. Definition of transitions and variables Petri Net Cassava Chips Production System

Definition of each transitions of The cassava chips production system		Definition of each variable of determine the time and length of the cassava chips production process	
t1	Cassava peeling and washing time	$v_{t1,k}$ = 20 minute	The length of the process of peeling and washing cassava
t2	Time of slicing cassava	$v_{t2,k}$ = 30 minute	The length of the process of slicing cassava
t3	Time of Soaking cassava slices	$v_{t3,k}$ = 60 minute	The length of the process of soaking cassava
t4	Time of drying cassava slices	$v_{t4,k}$ = 25 minute	The length of the process of drying cassava slices
t5	Time of frying cassava slices	$v_{t5,k}$ = 60 minute	The length of the process of frying cassava slices
t6	Time of draining cassava chips	$v_{t6,k}$ = 15 minute	The length of the process of draining cassava chips
t7	Time of sorting cassava chips	$v_{t7,k}$ = 25 minute	The length of the process of sorting cassava chips
t8	Time of seasoning cassava chips	$v_{t8,k}$ = 20 minute	The length of the process of seasoning cassava chips
t9	Time of packing	$v_{t9,k}$ = 60 minute	The length of the process of packing
t10	Time of distribution	$v_{t10,k}$ = 600 minute	The length of the process of distribution

The above variables are used to form the algebra of max plus model of system of the cassava chips production which is given below:

$t_1(k) = v_{t1,k} \otimes t_1(k-1)$
$t_2(k) = v_{t2,k} \otimes t_1(k) = v_{t2,k} \otimes v_{t1,k} \otimes t_1(k-1)$
$t_3(k) = v_{t3,k} \otimes t_2(k) = v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k} \otimes t_1(k-1)$
$t_4(k) = v_{t4,k} \otimes t_3(k) = v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k} \otimes t_1(k-1)$
$t_5(k) = v_{t5,k} \otimes t_4(k) = v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k} \otimes t_1(k-1)$
$t_6(k) = v_{t6,k} \otimes t_5(k) = v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k} \otimes t_1(k-1)$
$t_7(k) = v_{t7,k} \otimes t_6(k) = v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k} \otimes$
$t_1(k-1)$ $t_8(k) = v_{t8,k} \otimes t_7(k) = v_{t8,k} \otimes v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes$
$v_{t2,k} \otimes v_{t1,k} \otimes t_1(k-1)$
$t_9(k) = v_{t9,k} \otimes t_8(k)$
$= v_{t9,k} \otimes v_{t8,k} \otimes v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k} \otimes$
$t_1(k-1)$
$t_{10}(k) = v_{t10,k} \otimes t_9(k)$
$= v_{t10,k} \otimes v_{t9,k} \otimes v_{t8,k} \otimes v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes$
$v_{t1,k} \otimes t_1(k-1)$

The equations above can be formed matrix as follows:

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$$\begin{pmatrix} t_1(k) \\ t_2(k) \\ t_3(k) \\ t_4(k) \\ t_5(k) \\ t_6(k) \\ t_7(k) \\ t_8(k) \\ t_9(k) \\ t_{10}(k) \end{pmatrix} = \begin{pmatrix} A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ G & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ H & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ I & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ J & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{pmatrix} \otimes \begin{pmatrix} t_1(k-1) \\ t_1(k) \\ t_2(k) \\ t_3(k) \\ t_4(k) \\ t_5(k) \\ t_6(k) \\ t_7(k) \\ t_8(k) \\ t_9(k) \end{pmatrix}$$

With:

$A = v_{t1,k}$

$B = v_{t2,k} \otimes v_{t1,k}$

$C = v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

$D = v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

$E = v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

$F = v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

$G = v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

$H = v_{t8,k} \otimes v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

$I = v_{t9,k} \otimes v_{t8,k} \otimes v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

$J = v_{t10,k} \otimes v_{t9,k} \otimes v_{t8,k} \otimes v_{t7,k} \otimes v_{t6,k} \otimes v_{t5,k} \otimes v_{t4,k} \otimes v_{t3,k} \otimes v_{t2,k} \otimes v_{t1,k}$

For example,

$$\begin{aligned} v_{t1,k} &= 20\ minute \\ v_{t2,k} &= 30\ minute \\ v_{t3,k} &= 60\ minute \\ v_{t4,k} &= 25\ minute \\ v_{t5,k} &= 60\ minute \\ v_{t6,k} &= 15\ minute \\ v_{t7,k} &= 25\ minute \\ v_{t8,k} &= 20\ minute \\ v_{t9,k} &= 60\ minute \\ v_{t10,k} &= 600\ minute \end{aligned}$$

So,

$$\begin{pmatrix} t_1(k) \\ t_2(k) \\ t_3(k) \\ t_4(k) \\ t_5(k) \\ t_6(k) \\ t_7(k) \\ t_8(k) \\ t_9(k) \\ t_{10}(k) \end{pmatrix} = \begin{pmatrix} 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 50 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 110 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 135 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 195 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 210 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 235 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 255 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 315 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 915 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{pmatrix} \otimes \begin{pmatrix} t_1(k-1) \\ t_1(k) \\ t_2(k) \\ t_3(k) \\ t_4(k) \\ t_5(k) \\ t_6(k) \\ t_7(k) \\ t_8(k) \\ t_9(k) \end{pmatrix}$$

When the initial state

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$$\begin{pmatrix} t_1^*(0) \\ t_1(0) \\ t_2(0) \\ t_3(0) \\ t_4(0) \\ t_5(0) \\ t_6(0) \\ t_7(0) \\ t_8(0) \\ t_9(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

So that

$$\begin{pmatrix} t_1(k) \\ t_2(k) \\ t_3(k) \\ t_4(k) \\ t_5(k) \\ t_6(k) \\ t_7(k) \\ t_8(k) \\ t_9(k) \\ t_{10}(k) \end{pmatrix} = \begin{pmatrix} 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 50 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 110 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 135 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 195 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 210 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 235 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 255 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 315 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 915 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 20 \\ 50 \\ 110 \\ 135 \\ 195 \\ 210 \\ 235 \\ 255 \\ 315 \\ 915 \end{pmatrix}$$

If the production of cassava chips operates Monday to Saturday, starting at 08.00 WIB. Based on the results above, it shows that the first production on that day when t1 time for stripping and washing cassava is completed at 08.20 WIB, t2 time for slicing cassava is finished at 08.50 WIB, t3 time for soaking cassava slices is finished at 09.50 WIB, t4 time for drying cassava slices is complete. at 10.15 WIB, t5 when the cassava slices were fried finished at 11.15 WIB, t6 when the cassava chips were drained finished at 11.30 WIB, t7 the cassava chip sorting time was finished at 11.55 WIB, t8 the seasoning time for cassava chips was finished at 12.15 WIB, t9 the packaging time is finished at 13.15 WIB, and t10 the distribution time is finished at 23.15 WIB.

Thus, every day this cassava chip producer can complete its production from 08.00 to 23.15 WIB. According to the assumptions given, every day the producer processes cassava as much as 3000 grams per day. The cassava after being peeled, washed, sliced, dried and fried weighs 2100 grams. Cassava chips are packaged with each package weighing 25 grams/pack, in other words, producers distribute 84 packs of cassava chips every day.

4. Conclusions

The cassava chips production system can be graphed and a petri net can be formed using the Woped 3.1.0 tool which has 10 places which can be written as the set p = {p1, p2, p3, p4, p5, p6, p7, p8, p9, p10 } and has 10 transitions that can be written as the set t = {t1, t2, t3, t4, t5, t6, t7, t8, t9, t10} and analyzed so that a coverability tree can be built. These results are modeled into max – plus algebra and analyzed so that the first production of the day is obtained when t1 the time for stripping and washing cassava is completed at 08.20 WIB (if production starts at 08.00 WIB), t2 when slicing cassava is completed at 08.50 WIB, t3 the time of soaking the cassava slices was completed at 09.50 WIB, t4 the drying time of the cassava slices

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was completed at 10.15 WIB, t5 the time of the fried cassava slices was completed at 11.15 WIB, t6 when the cassava chips were drained was completed at 11.30 WIB, t7 the cassava chips sorting time was completed at 11.30 WIB to 11.55 WIB, t8 the seasoning time of cassava chips was completed at 12.15 WIB, t9 the packaging time was completed at 13.15 WIB, and t10 the distribution time was completed at 23.15 WIB. Thus, every day this cassava chip producer can finish its production from 08.00 to 23.15 WIB. Suggestions for further research, research can be carried out with other methods related to determining the optimal time of the cases discussed in this research.

Author Contributions

First Author contributed to Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Project administration; Software; Validation; Visualization; Roles/Writing - original draft. First author and second author were searching reference resources together. Second author with third author were Funding acquisition; Writing - review & editing, especially translate from Indonesia language to English language together. In Addition, Second author contributed also to make reference in mandeley and enter the manuscript in the username of the first author.

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Declaration of Competing Interest

The reseacher inform that they are not aware of any competing financial interests or personal relationships that may affect the work presented in this article.

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