APPLICATION RECURSIVE LEAST SQUARE FOR ANALYSIS CLOSED LOOP CIRCUIT OP AMP

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Abstrak

Tujuan dari makalah ini adalah pengembangan dari pendekatan input-output bersama untuk identifikasi sistem loop tertutup dari suatu rangkaian penguat. Metode RLS (Recursive Least Square) digunakan untuk mengestimasi parameter yang belum diketahui dari sistem tersebut. Rangkaian untuk percobaan ini adalah orde 2. Input untuk percobaan adalah sinus sinyal dari fungsi generator. Input dan output dari percobaan di rekam oleh kartu ADC yang terkoneksi pada komputer. Dari keluaran hasil estimasi menggunakan RLS kita mendapatkan data dari pengukuran dan hasil estimasinya secara umum sama. Kesalahan dalam proses estimasi yaitu mendekati nilai nol.
Kata kunci : PRBS, Recursive Least Square, *sistem loop tertutup*

Abstract

The aim of the paper is development of a joint input-output approach for the identification of closed loop system in the case off circuit amplifier. A recursive least square (RLS) method is used to estimate the unknown parameters of the system. The circuit for this experiment is second order. The input for experiment is sine signal from function generator. The input and output from experiment is recorded with ADC card 0connected computer. From the result output estimation with recursive least square we get the data from measurement and the estimation are generally same. Error in process estimation is quickly near zero value.
Key words : PRBS, Recursive Least Square, Closed loop amplifier

1. INTRODUCTION

The closed-loop system identification approaches can be divided into three main groups: a direct approach, an indirect approach, and joint input-output, which are worked out to identify the open-loop system. The direct approach is realized, using the input and noisy output observations when the feedback is ignored. In such a case, the open-loop system is identified if the respective identifiability conditions are satisfied . The indirect approach is used, first to indentify some closed-loop system transfer function and second to determine the open-loop system parameters, assuming that the regulator is known beforehand.

The direct frequency estimation from measured sinusoids plays an important role in signal processing unit of communication, control, instrument and power systems. In many cases, the efficiency and stability of given system is influenced with the accurate and reliable frequency measurement.

System identification has gain much interest in many engineering applications where the parameter of the system can be estimated using recursive and non-recursive manner such as least squares, recursive least square, recursive instrumental variables and recursive maximum likelihood methods. System identification can be divided into nonparametric and parametric estimation methods. For nonparametric method such as transient and correlation analysis, the result is easy to obtain but the derived model will be rather inaccurate and sensitive to noise, whilst parametric is an estimation method based on use-specified models or ready-made models to estimate the model and give an accurate results.

System identification problem can be classified as grey box, black box and white box modeling. Black box models approach can be used which allow sufficient input-output measurements in developing a mathematical model that represent the system dynamics.

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In the construction of model, physical laws such as Newton's Law can be used but required specialize knowledge in the system. The model which is derived directly from some first principles by taking into account the connection between the components of the system is called a white box model. A grey box model or semi-physical modeling are obtained when white box model contains some parameter that have unknown or uncertain numerical values. Meanwhile, for nonlinear model identification, Kaddisi and Jelali have described grey box modeling by assuming a model structure is obtained from first principle of hydraulic system and used recursive method for estimating the model parameters.

This study describes recursive identification in discrete time using least square algorithm for closed loop amplifier system second order.

2. CLOSED LOOP AMPLIFIER

Operational amplifier can have either a closed-loop operation or an open-loop operation. The operation (closed-loop or open-loop) is determined by whether or not feedback is used.

2.1. Transfer Function

Transfer functions are commonly used in the analysis of systems such as single-input single-output filters, typically within the fields of signal processing, communication theory, and control theory. The term is often exclusively to refer to linear, time-invariant systems (LTI), as covered in this article. Most real systems have non-linear input/output characteristics, but many systems, when operated within nominal parameters have behavior that is close enough to linear that LTI system theory is an acceptable representation of the input/output behavior.

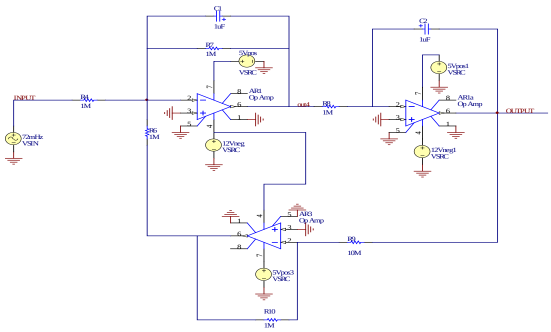

Figure 2-1. Closed-loop Circuit Op-amp

The transfer function of Figure 2-1 we can get like equation 2-1.

$$L(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + s + 1} \qquad (2-1)$$

2.2. Model Identification

System identification involves creating a model for the system in real application, given the same input as the original system, the model will produce an output that matches the original system output to a certain degree of accuracy. The input or excitation to the system and model, and their corresponding output are used to create and tune that model until a satisfactory degree of model accuracy is reached. As shown Figure 2-2, the input $u(t)$ is fed to both the system and the model $M(\hat{\theta})$, then their corresponding outputs $y(t)$ and $\hat{y}(t, \hat{\theta})$, are produced and matched. The error $e(t)$ reflects how much the model matches the system, the lower the error, the more the model resembles the system.

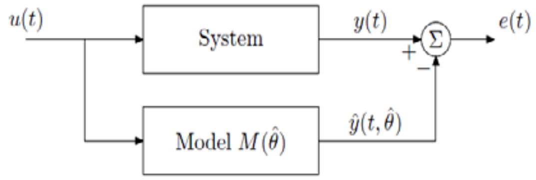

Figure 2-2. Model Identification

2.3. PRBS

A Pseudo-Random Binary Sequence, also known as Maximal Length Sequence (MLS), is a periodic, deterministic signal properties similar to white noise. We can generate a pseudo-random binary sequence using an *n*-bit shift register with feedback through an exclusive-OR function. While appearing random, the sequence actually every 2*n*-1 values.

When using a whole period, the psudo-random binary sequence has special mathematical advantages that make ir attractive as a stimulus signal. in particular, we can attribute variations in response signal between two periods of the stimulus to noise due to the periodic nature of the signal. Figure 2-3 show the picture of pseudo-random binary sequence.

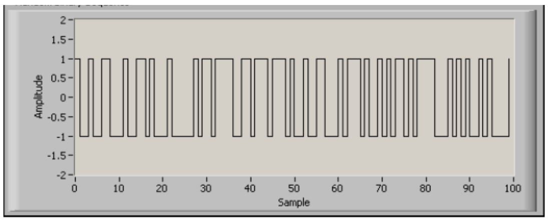

Figure 2-3. Pseudo-random binary sequence

3. SYSTEM IDENTIFICATION

System identification is a field of modeling dynamic system from measured data using mathematical algorithm. The identification process consists of estimating the unknown parameters of the systems dynamics. The identification method based on least square algorithm has been recommended for the identification process for ease of implementation and application in real systems. For the linear identification process, discrete time ARX model for the many experiment linear system had been used. The ARX model for the linear system is given as shown in equation (3-3) [4].

$$A(q^{-1})y(k) = q^{n_k}B(q^{-1})u(k) + e(k) \qquad (3-1)$$

where

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_{n_a}q^{-n_a},$$
$$B(q^{-1}) = b_1 + b_2q^{-1} + \cdots + b_{n_b}q^{-n_b+1}$$

and the symbol *q-1* denotes the backward shift operator, *u(k)* and *y(k)* are the system input and output, respectively and *e(k)* is the white noise of the system with zero mean. The parameter a_i and b_i are real coefficient while n_a is the number of poles, n_b is the number of *b* parameters or equal to the number of zeros plus1. n_k is the number of samples before the input affects output of the system where also called the delay or dead time of the model.

The disadvantage of off-line system identification is the need to acquire a sufficient set of experimental test data of the system, which may require a long time and large efforts. Besides, data recollection and estimation process must be conducted if any modification on the system is applied. Therefore, online identification saves time in data collection and improves the model accuracy and reliability. Furthermore any changes in the system components and structure will be affected the system model without the need to data recollection. Equation (3-1) can be put into linear regression form as follows as shown in equation (3-2).

$$y(k) = \varphi^T(k)\theta(k) + e(k) \qquad (3-2)$$

where

$$\varphi(k) = (-y(k-1), \dots, -y(k-n_a), u(k-n_k), \dots u(k-n_b-n_k+1))^T$$

$$\theta(k) = (a_1, a_2, \dots, a_{n_a}, b_1, b_2, \dots, b_{n_b})$$

3.1. Off-line parameter interval estimates

Among various algorithms for plant parameter estimation and estimation we find the popular least square (LS) and the recursive least squares (RLS) and its relatives. The LS and RLS algorithm will be the point of departure in this paper. Let the 'true' process be

$$y(t) = \theta^T\varphi(t) + v(t) \qquad (3\text{-}3)$$

Where $\theta = (a_1 \dots a_n\ b_1 \dots b_m)^T$ is the parameter vector and $\varphi(t) = [-y(t-1) \dots -y(t-n)\ u(t-1) \dots u(t-m)]^T$ is the vector of measured lagged input-output data. $\{u(t)\}$ is the measured input signal and $\{y(t)\}$ is the measured output signal. t = 1,2,3… is the running sample index. The equation error $\{v(t)\}$ includes all effects of measurement noises, mismodelling, disturbances and other uncertainties in the given description of the data. Then the LS estimates $\hat{\theta}^{LS}(N)$ at $t = N$ is given by

$$\hat{\theta}^{LS}(N) = \left(\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\,\varphi(t)^T\right)^{-1}\frac{1}{N}\sum_{t=1}^{N}\varphi(t)y(t) \qquad (3\text{-}4)$$

Introduce the matrix

$$R(N) = \frac{1}{N}\sum_{t=1}^{N}\varphi\,(t)\varphi^T \qquad (3\text{-}5)$$

The pure RLS estimate $\hat{\theta}(t)$ is given by

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\varphi(t)\left[y(t) - \hat{\theta}(t-1)^T\varphi(t)\right] \qquad (3\text{-}6)$$

$$P(t) = P(t-1) - \frac{P(t-1)\varphi(t)^T P(t-1)}{1+\varphi(t)^T P(t-1)\varphi(t)} \qquad (3-7)$$

For suitable initial conditions of the RLS algorithm $\hat{\theta}(t) = \hat{\theta}^{LS}(t)$ and $P(t) = R(t)^{-1}/t$ for all t; for any positive definite $P(0)$, $\hat{\theta}(t)$ and $P(t)$ converge to $\hat{\theta}^{LS}(t)$ and $R(t)^{-1}/t$ respectively. For constant θ the estimate $\hat{\theta}(t)$ converges to θ under ideal conditions. Also under ideal conditions $P(t)$ is the normalized variance of the estimate $\lambda_0 P(t) = E\{[\hat{\theta}(t) - \theta]\,[\hat{\theta}(t) - \theta]^T\}$, where λ_0 is the variance of $v(t)$.

Like the pure RLS estimator, most algorithm give point and variance estimates only. Under ideal conditions the variance matrix $P(t)$ of the RLS estimator could be used for estimating a likely parameter set. With $V(t)$ or V known. It is now be shown how this assumption may be used to compute parameter interval bounds. It is easy to show that the LS estimation error $\hat{\theta}(N) = \hat{\theta}^{LS}(N) - \theta$ is given by

$$\bar{\theta}(N) = [R(N)]^{-1}\frac{1}{N}\sum_{t=1}^{N}\varphi\,(t)v(t) \qquad (3-8)$$

Comparing with (3-1), notice that with $\{v(t)\}$ given, (3-6) can be implemented in the recursive form (3-3), (3-4), with $v(t)$ replacing $y(t)$ and $\bar{\theta}$ replacing $\hat{\theta}$ in (3-3), with $P(t)$ given by (3-4):

$$\tilde{\theta}(t) = \tilde{\theta}(t-1) + P(t)^T\varphi(t)[v(t) - \tilde{\theta}(t-1)^T\varphi(t) \qquad (3\text{-}9)$$

However, the method is based on off-line system identification of a particular system and not able to detect for systems changes during the operations, such as the effects of temperature, supply pressure changes and aging.

3.2. On-line parameter interval estimates

Recursive least square (RLS) method has been widely used and it has several advantages such as easy numerical solution and fast parameter convergence [4]. The method also gives consistent modeling accuracy over a wide range of operating conditions and seems best linear unbiased estimate. The speed of parameter convergence depends on the forgetting factor used. Faster parameter convergence can be obtained if the value of forgetting factor is reduced, but this leads to noise implication [2]. For the RLS algorithm to be able to update the parameters at each sample time, it is necessary to define an error from equation (3-4):

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1) \qquad (3-10)$$

$$P(k) = \frac{1}{\lambda}P(k-1)\left[I_P - \frac{\varphi(k)\varphi^T P(k-1)}{\lambda + \varphi^T(k)P(k-1)\varphi(t)}\right] \qquad \text{(3-11)}$$

$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\varphi(k)\varepsilon(k)$ (11)

Where $\hat{\theta}(t)$ is the estimated parameter, *P(t)* is the covariance matrix, the subscript *'p'* is the dimension of the identity matrix, $p = na + nb$, and λ is the forgetting factor, $0 < \lambda \leq 1$.

Direct identification, indirect identification and joint input-output identification methods can be performed in the closed-loop identification problem [6]. The direct identification method in closed-loop system is used to identify the system unknown parameters since this method is simple and applicable without taking account of the presence of a feedback controller. This approach is especially suitable for system with nonlinear or unknown feedback mechanisms. Indirect identification is based on the assumption that the feedback control law is known. Then, the closed-loop system is identified and the open-loop system can be determined using the identified closed-loop system and the known control structure. Joint input-output identification is performed using the acquire input and output as outputs of a multivariable system in response to an external signal such as noise. Open-loop system parameters are obtained using the identified multivariable system.

4. EXPERIMENT AND SIMULATION

In this experiment we use function generator as the input of sine signal. The circuit for this experiment is closed loop amplifier that shown in Figure 2-1. The output and input for this experiment recorded by computer use ADC card. The setup for this experimentdepicted in Figure 4-1.

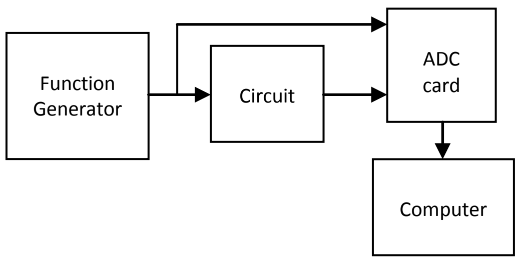

Figure 4-1. Set-up for Experiment

4.1. Real Experiment

Amplitude for the input sine signal is ± 3.5 volt. Vpp from input signal is 7 volt. The output from circuit is fall on to Vpp 6.6 volt. So the signal reduce 0.4 volt. Plotting data input and output from this experiment depicted in Figure 4-2. The red signal is input and the blue signal is output. There are 2000 data in this experiment so it is so long, if we take several data and then plotted depicted at Figure 4-1. Because the input signal is constant and continues in the amplitude and frequency several data is enough to show signal input and output from measurement.

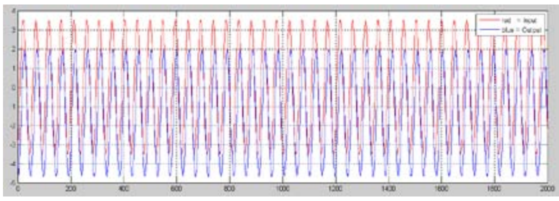

Figure 4-2. Input and output from experiment

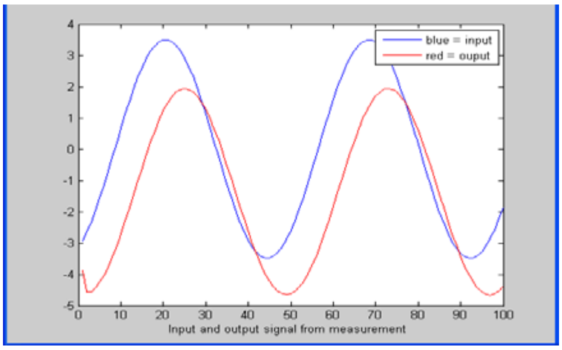

Figure 4-3. Several data Input and output experiment

4.2. Estimation output with RLS

The recursive least squares (RLS) algorithm has a long tradition of use in the area of system identification and modeling.

In this section is shown a simulation example to illustrate the main characteristics of the RLS algorithm proposed in this paper.

A. Simulation Result

The closed loop system is described by

$$y(k) = -a1*y(k-1) - a2*y(k-2) + b1*u(k-1) + b2*u(k-2)$$

Parameter a1, a2, b1 and b2 can be founded in process RLS. From Figure 4-3 we can see that the output data for the first part signal is not the same with the other signal. After several data input the output is constant as sine signal that shown in Figure 4-2. The estimation tetha is depicted in Figure 4-4.

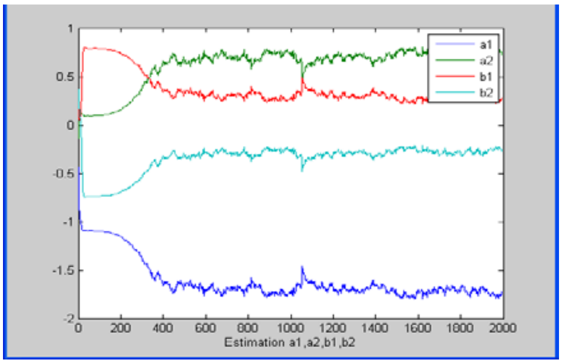

Figure 4-4. Estimation Parameter

The result of output estimation from this simulation can be depicted in Figure 4-5. From this figure we can see that the output from estimation and measurement are generally same. But the output estimation under 5th data is not same with the output from measurement.

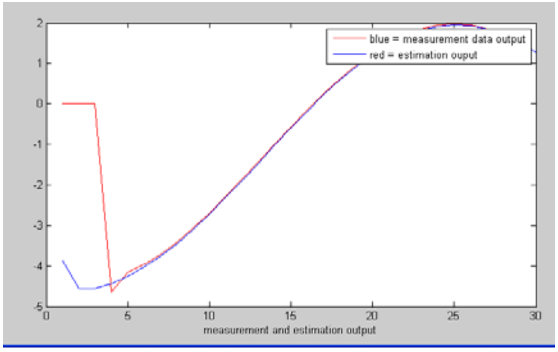

Figure 4-5. Ouput measurement and estimation

The error from processing estimation is depicted in Figure 4-6. After data 5th the error is directly to zero.

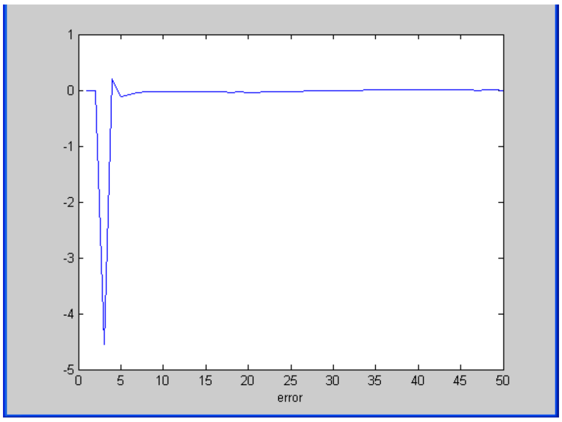

Figure 4-6. Error in estimation process

5. DISCUSSION

It is seen from figure 2-1 that the transfer function of the closed loop amplifier presented in equation (2-1) is a 2nd-orde system. To be concluded from the transfer function drawn in the curve of figure 4.3 that the system is stable. This conclusion of stabilization is taken from the fact that the real parts of the roots of the transfer function denumerator are all negative values. There were evidently two roots obtained from the 2nd-orde system. For the closed loop system presented it is got the real parts of the roots are all minus half. The result was easily solved from the denumerator of transfer function equation. This denumerator is usual named characteristic equation of the system. Equalizing the characteristic equation of the system to zero brings the solution of the system stability. Visually measurement of determining stability can be established by giving entrée of dirac signal to the system and then observing the output result. Response of the dirac denotes whether the system is stable or not. Raising amplitude for infinite time signifies that the system is unstable and vice versa. Then for the system presented in the paper it was concluded that the system was stable.

6. CONCLUSION

This paper presented circuit operational amplifier closed loop identification using recursive least square. The input from experiment is sine signal with the amplitude and frequency constant. The response output from the circuit is also sine signal. From the first output data less than 5th data the output is not same with the next output, this case is depicted in Figure 4-1. After estimation with recursive least square we get the estimation generally same with the output data from measurement. The error in processing estimation is quickly to near zero.

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HASIL DISKUSI DALAM PELAKSANAAN SEMINAR

Pertanyaan :

1. Apakah bedanya OP AMP dengan amplifier biasa ? (Sutisno, Pustekroket)
2. Di dalam rangkaian closed loop, apa bedanya rangkaian amplifier dengan osilator. (Andreas P A, Pustekroket)

Jawaban :

1. OP AMP mempunyai sifat khusus dibandingkan dengan amplifier biasa. Sifat itu adalah : mempunyai penguatan yang tinggi atau infinite, impedansi masukan tinggi sampai orde mega ohm, dan mempunyai dua terminal masukan yaitu inverting input dan non-inverting input.
2. Dalam rangkaian close loop terdapat rangkaian dan komponen balikan (feedback). Untuk rangkaian amplifier, balikan ini merupakan negative feedback, sedangkan untuk rangkaian osilator balikan merupakan positive feedback.

Appendix

Code Program

```
% y(k)=-a1*y(k-1)-a2*y(k-2)+b1*u(k-1)+b2*u(k-2)
% the first column of the data you sampled is input and the next column is output.
% chun jie kuai le
clc
clear all
close all

%--------------------------------- load file data -------------------------
load fendi.txt % load file endi.txt
fendi;

number = size(fendi);
N = number(1,1);
u = fendi(:,2); %  separate file fendi to input variable
y = fendi(:,1); %  separate file fendi to output variable

figure(1);
l = 1:100;
plot (l,u(1:100),'b',l,y(1:100),'r');
legend('blue = input','red = ouput ')
xlabel('Input and output signal from measurement ')

figure(2);
plot(u,y);
xlabel('input VS output from experiment');

%-------------------------------- initialisai value
y_heda = [];
lamda = 0.98;
w_heda = 0.001*rand(1,4); % 1. Initialize the parametric vector using a small positive number ?.
pshi_heda = zeros(1,2000)'; % 2. Initialize the data vector phi.
P =  diag([w_heda]); % 3. Initialize the k × k matrix P(0).
pshi = [];
tetha_heda = zeros(4,1);
P0 = 1e2*eye(4);
```

```
tetha_heda1 = [];
 for k = 3:1:2000

   for i=1:2
      u1(i) = u(k-i);
   end
   for i=1:2
      y1(i) = y(k-i);
   end

   pshi = [-y1'; u1'];

   sample_out = y(k);
   y_heda(k)  = pshi' * tetha_heda;
   error(k)   = y(k) - y_heda(k);
   P0         = (P0 - ((P0 * pshi * pshi' * P0) / (lamda + (pshi' * P0 * pshi)))) / lamda;
   tetha_heda = tetha_heda + (P0 * pshi * (sample_out - (pshi' * tetha_heda)))
   Recordedw(1:4,k) = tetha_heda;

end
%pshi
tetha_heda
figure(3);
i = 1:30;
plot(i,y_heda(1:30),'r',i,y(1:30),'b');
legend('blue = measurement data output','red = estimation ouput');
xlabel('measurement and estimation output');

figure(4);
plot(error(1:50));
xlabel('error');

figure(5);
plot(Recordedw(1:4,4:2000)');
xlabel('Estimation a1,a2,b1,b2');
legend('a1','a2','b1','b2');
```