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WATER TEMPERATURE MODELING OF LAKE RANAU, SUMATERA ISLAND

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ABSTRACT

The Lake Ranau lies at 538 m above sea level, located in South Sumatera and Lampung Provinces. The maximum depth of the lake is 220 m and volume is 19.92×10^9 m ^{3}. Lake temperature influences the transportation and transformation of chemical mass, phytoplankton and detritus in the water body. We model the water temperature by applying the derivative equation of heat-conductive Fourier and the momentum and continuity current 2D multi layer from Navier-Stokes equation. To solve the equations we use finite difference method and numerical scheme with Alternating Direction Implicit (ADI). The wind, shoreline and bottom morphology are considered as a boundary condition. While the current velocity from rivers flow can be ignored. The model reveals that current pattern forms eddy current at each layer, and temperature distribution tend to be similar to the observation especially arround hot water inflow sites.

Key words: Lake Ranau; modelling; water temperature

1. INTRODUCTION

Lake Ranau is located at an altitude of 538 m above sea level, and administratively belongs to the Provinces of South Sumatera and Lampung. Recently, the lake is utilized as source of Ogan-Komering irrigation beside as recreational and potential area for fisheries.

Lake Ranau is a tectono-vulcanic lake. Before extensive eruption of Ranau volcanic mount, this area was a graben, it is difficult to determine the part that was subsided during caldera formation (Bemmelen, 1949). From its rectangular shape that both sides are parallel to the Sumatra fault, it could be interpreted as an old pull-apart area, which is no longer active. Lake Ranau structure type is not a caldera but it is a pull-apart lake. Analysis showed that pull-apart area have maximum normal tension distribution, which indicate that this area is potentially a rupture. Mount Seminung at the southern part of the lake proved that magmatic activity occurred as the result of pulling during pull-apart process (Primastuti, 1994).

The Research and Development Center for Limnology conducted data collection for limnological study that is to be utilized as data-base in the development and restoration of Lake Ranau. Current in a lake water body is an important parameter for limnological process. Chemical mass transport, distribution of nutrient, phytoplankton and detritus in the water body are highly influenced by the lake water current. Lake current is very difficult to measure because of direction variation and the weakness the current. Therefore, current pattern estimation occurred in Lake Ranau had been estimated from temperature distribution pattern in the lake water body since water mass density is strongly influenced by the temperature.

2. MATERIAL AND METHOD

Lake Ranau current pattern estimation can be carried out based on lake bathymetry gained from bathymetric measurement. The bathymetric measurement had been carried out by using a boat, ODOM Echotracht DF 3200 MK II echo sounder, and DGPS Garmin Srvy II for the purpose of positioning. Based on the morphometry of measurements result, segmentations were made by specified intervals as domain of computation.

Current estimation of Lake Ranau water body was done by equation of movement and two-dimension layered liquid continuity, which is generated from Navier-Stokes equation. Generation of the equation is as follows:

– Water movement equation in x axis direction for surface layer,

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y}-\frac{(u_{3/2}-u_1)w_{3/2}}{h_1-\zeta}$$

$$=f.v_1-\frac{1}{\rho_1}\frac{\partial p_1}{\partial x}+A_x\frac{\partial^2 u_1}{\partial x^2}+A_y\frac{\partial^2 u_1}{\partial y^2}-\frac{\gamma_i^2(u_1-u_2)\sqrt{(u_1-u_2)^2+(v_1-v_2)^2}}{h_1-\zeta} \qquad \text{eq. 1}$$

– Water movement equation in y axis direction for surface layer,

$$\frac{\partial v_1}{\partial t}+u_1\frac{\partial v_1}{\partial x}+v_1\frac{\partial v_1}{\partial y}-\frac{(v_{3/2}-v_1)w_{3/2}}{h_1-\zeta}$$

$$=f.u_1-\frac{1}{\rho_1}\frac{\partial p_1}{\partial y}+A_x\frac{\partial^2 v_1}{\partial x^2}+A_y\frac{\partial^2 v_1}{\partial y^2}-\frac{\gamma_i^2(v_1-v_2)\sqrt{(u_1-u_2)^2+(v_1-v_2)^2}}{h_1-\zeta} \qquad \text{eq. 2}$$

Where $p_1=g.\rho_1.\zeta+0{,}5.g.\rho_1.h_1$

– Water movement equation in x axis direction for intermediate layer

$$\frac{\partial u_k}{\partial t}+u_k\frac{\partial u_k}{\partial x}+v_k\frac{\partial u_k}{\partial y}+\frac{\left(u_{k-1/2}-u_k\right)w_{k-1/2}-\left(u_{k-1/2}+u_k\right)w_{k+1/2}}{h_k}$$

$$=f.v_k-\frac{1}{\rho_k}\frac{\partial p_k}{\partial x}+A_x\frac{\partial^2 u_k}{\partial x^2}+A_y\frac{\partial^2 u_k}{\partial y^2}$$

$$+\frac{\gamma_i^2\left(u_{k-1}-u_k\right)\sqrt{\left(u_{k-1}-u_k\right)^2+\left(v_{k-1}-v_k\right)^2}}{h_k}-\frac{\gamma_i^2\left(u_k-u_{k+1}\right)\sqrt{\left(u_k-u_{k+1}\right)^2+\left(v_k-v_{k+1}\right)^2}}{h_k} \qquad \text{eq. 3}$$

- **Water movement equation in y axis direction for intermediate layer**

$$\frac{\partial v_k}{\partial t}+u_k\frac{\partial v_k}{\partial x}+v_k\frac{\partial v_k}{\partial y}+\frac{\left(v_{k-1/2}-v_k\right)w_{k-1/2}-\left(v_{k-1/2}+v_k\right)w_{k+1/2}}{h_k}$$

$$=f.u_k-\frac{1}{\rho_k}\frac{\partial p_k}{\partial x}+A_x\frac{\partial^2 v_k}{\partial x^2}+A_y\frac{\partial^2 v_k}{\partial y^2}$$

$$+\frac{\gamma_i^2\left(v_{k-1}-v_k\right)\sqrt{\left(u_{k-1}-u_k\right)^2+\left(v_{k-1}-v_k\right)^2}}{h_k}-\frac{\gamma_i^2\left(v_k-v_{k+1}\right)\sqrt{\left(u_k-u_{k+1}\right)^2+\left(v_k-v_{k+1}\right)^2}}{h_k} \qquad \text{eq.4}$$

Where $\rho_k = g.\rho_1.\zeta+\sum_{i=1}^{k-1} g\rho_i h_i+0.5 g\rho_k h_k$

- **Water movement equation in x axis direction for bottom layer**

$$\frac{\partial u_k}{\partial t}+u_k\frac{\partial u_k}{\partial x}+v_k\frac{\partial u_k}{\partial y}+\frac{\left(u_{k-1/2}-u_k\right)w_{k-1/2}}{h_k}$$

$$=f.v_k-\frac{1}{\rho_k}\frac{\partial p_k}{\partial x}+A_x\frac{\partial^2 u_k}{\partial x^2}+A_y\frac{\partial^2 u_k}{\partial y^2}$$

$$+\frac{\gamma_i^2\left(u_{k-1}-u_k\right)\sqrt{\left(u_{k-1}-u_k\right)^2+\left(v_{k-1}-v_k\right)^2}}{h_k}-\frac{\gamma_i^2 . u_k\sqrt{u_k^2+v_k^2}}{h_k} \qquad \text{eq. 5}$$

- **Water movement equation in y axis direction for bottom layer**

$$\frac{\partial v_k}{\partial t}+u_k\frac{\partial v_k}{\partial x}+v_k\frac{\partial v_k}{\partial y}+\frac{\left(v_{k-1/2}-v_k\right)w_{k-1/2}}{h_k}$$

$$=f.u_k-\frac{1}{\rho_k}\frac{\partial p_k}{\partial x}+A_x\frac{\partial^2 v_k}{\partial x^2}+A_y\frac{\partial^2 v_k}{\partial y^2}$$

$$+\frac{\gamma_i^2\left(v_{k-1}-v_k\right)\sqrt{\left(u_{k-1}-u_k\right)^2+\left(v_{k-1}-v_k\right)^2}}{h_k}-\frac{\gamma_i^2 . v_k\sqrt{u_k^2+v_k^2}}{h_k} \qquad \text{eq. 6}$$

Where, $\rho_k = g.\rho_1.\zeta + \sum_{i=1}^{k-1} g.\rho_i.h_1 + 0{,}5g.\rho_k.h_k$

- Continuity equation of surface layer,
$$\frac{\partial \zeta}{\partial t} + \frac{\partial\{u_1(h_1+\zeta)\}}{\partial y} + \frac{\partial\{v_1(h_1+\zeta)\}}{\partial y} - w_{1/2} = R_{in} - R_e$$

- Continuity equation of intermediate layer,
$$\frac{\partial(u_k h_k)}{\partial x} + \frac{\partial(v_k h_k)}{\partial y} + w_{k-1/2} - w_{k+1/2} = 0$$

- Continuity equation of bottom layer
$$\frac{\partial(u_k h_k)}{\partial x} + \frac{\partial(v_k h_k)}{\partial y} + w_{k-1/2} = 0$$

T = time (second);
h_k = thickness of k layer (m);
ς = water level of average water level (m);
u_k and v_k = averages of water current velocity for x and y axis direction (m/sec);
$u_{k+1/2}$ = velocity of water current between k and k+1 layer for x axis direction (m/sec);
$v_{k+1/2}$ = velocity of water current between k and k+1 layer for y axis direction (m/sec);
$w_{k+1/2}$ = velocity of vertical water current between k and k+1 layer (m/sec);
water density on the k water layer (kg/m^3) ;
p_k = water tension on the k layer ($kg/m\text{-}sec^2$) ;
A_x and A_y = Eddy momentum diffusion in x and y axis direction (m^2/sec) ;
γ_i = friction coefficient between layer ;
γ_b = friction coefficient on lake bottom;
f = coriolis factor (1/sec) ;
R_{in} = rate of lake water surface increase resulted from water inlet and rainfall (m/sec);
R_e= rate of lake water surface decrease by evaporation (m/sec).

In this research, the water temperature corrects the density of the lake water. The correction is as follows:

$$\rho_k = a_0 + a_1(T_k - 273) + a_2(T_k - 273)^2 + a_3(T_k - 273)^3 + a_4(T_k - 273)^4 + a_5(T_k - 273)^5 \qquad \text{eq. 7}$$

Where a_0 - a_5 = constants, while T = water temperature on k layer (°K) that is estimated by heat balance equation as follows,

– Surface layer

$$\frac{\partial T_1(h_1+\zeta)}{\partial t}=\frac{\partial(M_{x1}T_1)}{\partial x}-\frac{\partial(M_{y1}T_1)}{\partial y}+\frac{\partial}{\partial x}\left\{K_{x1}(h_1+\zeta)\frac{\partial T_1}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{K_{y1}(h_1+\zeta)\frac{\partial T_1}{\partial y}\right\}$$
$$-K_z(T_1-T_2)+w_{\frac{1}{2}}T^{*}+\frac{1}{\rho_w c_p}\left(q_0-q_{\frac{1}{2}}\right)-\frac{1}{\rho_w c_p}(q_e+q_c+q_r)+R_{in}T_{in} \qquad \text{eq. 8}$$

– Intermediate layer

$$\frac{\partial T_k h_k}{\partial t}=\frac{\partial(M_{xk}T_k)}{\partial x}-\frac{\partial(M_{yk}T_k)}{\partial y}+\frac{\partial}{\partial x}\left\{K_{xk}h_k\frac{\partial T_k}{\partial x}\right\}+\frac{\partial}{\partial y}\left(K_{yk}h_k\frac{\partial T_k}{\partial y}\right)$$
$$-K_z(T_{k-1}-T_k)-K_z(T_k-T_{k+1})-w_{k-\frac{1}{2}}T^{*}+w_{k-\frac{1}{2}}T^{*}+\frac{1}{\rho_w c_p}\left(q_{k-\frac{1}{2}}-q_{k+\frac{1}{2}}\right) \qquad \text{eq. 9}$$

– Bottom layer

$$\frac{\partial T_k h_k}{\partial t}=\frac{\partial(M_{xk}T_k)}{\partial x}-\frac{\partial(M_{yk}T_k)}{\partial y}+\frac{\partial}{\partial x}\left\{K_{xk}h_k\frac{\partial T_k}{\partial x}\right\}+\frac{\partial}{\partial y}\left(K_{yk}h_k\frac{\partial T_k}{\partial y}\right)$$
$$-K_z(T_{k-1}-T_k)-w_{k-\frac{1}{2}}T^{*}-\frac{1}{\rho_w c_p}\left(q_{k-\frac{1}{2}}-q_{k+\frac{1}{2}}\right) \qquad \text{eq. 10}$$

M_{xk} and M_{yk} = integrated vertical water current velocity on x and y direction (m^2/det);
K_x and K_y = Eddy heat diffusion (m^2/sec);
K_z = vertical heat mixing coefficient (m/sec);
q_0 = isolation quantity on water surface without surface reflection (J/m^2.det);
$q_{k+\frac{1}{2}}$ = isolation quantity on the layer between k and k+1(J/m^2.sec);
q_e = loss of heat on lake water surface by evaporation (J/m^2.sec);
q_c = loss of heat on lake water surface by thermal conductivity (J/m^2.sec);
q_x = loss of heat on lake water surface by heat radiation (J/m^2.sec) ;
T_{in} = temperature of water entering the lake (° K);

ρ_w = water density (kg/m^2) ;
c_p = water specific heat (J/kg.°K) ;
$w_{k+\frac{1}{2}} \rangle 0$ than T^{*} = T_{k+1} and $w_{k+\frac{1}{2}} \langle 0$ than T^{*} =T_k

The solution of the above equations was found by finite difference method, where the numeric scheme was utilizing Alternating Direction Implicit (ADI) method (Kreyszig, 1988). In order to get established solution and to avoid floating error, the time step calculation was estimated by the formulae $\Delta t \le \frac{\Delta x}{\sqrt{2gh_{max}}}$, where Δt = step of calculation time (sec), Δx = length of segment interval (m), g = gravitational acceleration (m/sec^2), h_{max} = maximum lake depth (m).

The boundary condition for the solution of the above equations are segments of lake water body gained from bathymetric map of the lake, water quantity of inlet and outlet the lake that is estimated by float method and the temperature that is measured by logger thermo water. Computation was carried out by Fortran programming software on PC with Pentium III 550 MHz processor and 128 MB memories

3. RESULT AND DISCUSSION

From bathymetric measurement as presented on Figure 1, it is known that the water surface altitude is 538 msl, Lake Ranau surface area of 128.13 km^2, water volume of 19,92 x 10^9 m^3, and maximum depth of 221 m. Based on morphometry, segmentations were made by x, y segment intervals of 500 m, as presented on Figure 2.

Based on maximum depth, gravity of 9,8 m/sec^2, bottom friction coefficient of 0,0026 and program running trials and errors, it was resulted computation with time intervals (Δt) of 7 sec. Layer thickness were determined: 1, 1, 2, 2, 2, 2, 2, 5, 5, 5, 5, 5, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 25, 25,

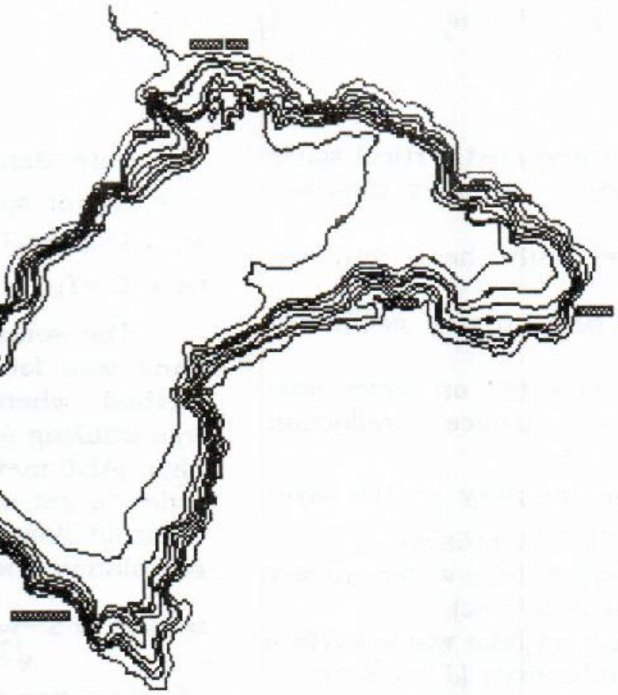

Fig. 1. Bathymetric map of Lake Ranau

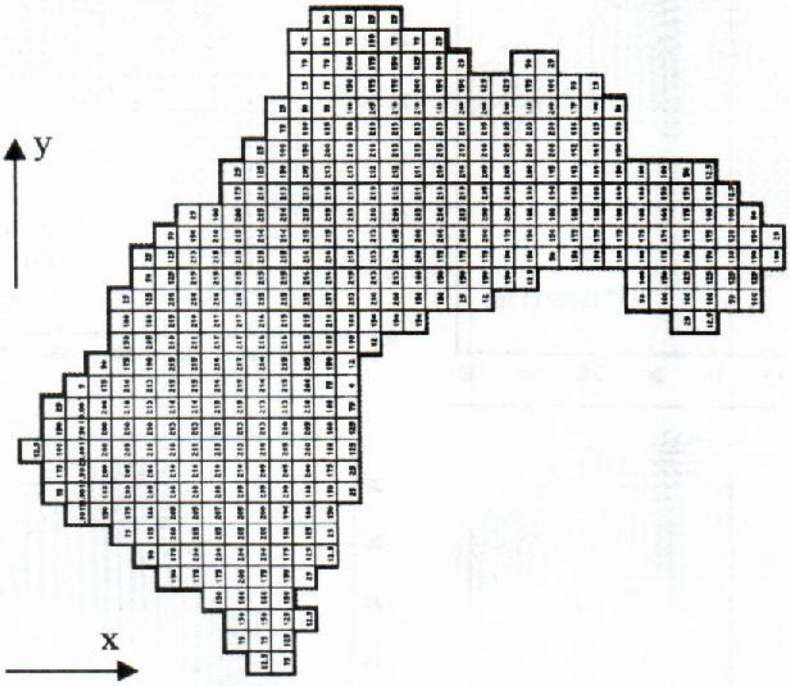

Fig. 2. Segmentation of Lake Ranau were made by x, y segment intervals of 500 m

25, 25, 25 m with friction coefficient between layer of 0,001. By 5th time iterations, the current pattern of Lake Ranau for each layer as presented on Figure 3.

Based on integrated vertical water current velocity on x and y direction, Eddy heat diffusivity, vertical heat mixing coefficient, isolation quantity, and loss heat on lake water surface, area and vertical distribution of temperatur have been calculated. The area temperature distribution of Lake Ranau presented on Figure 4, whereas the vertical distribution on Figure 5.

From Figure 3, it is known the model reveals that the current pattern forms eddy current at each layer, and inlet current has no effect on current pattern. Current pattern of Lake Ranau is more effected by coriolis factor, water density, basin morphology, and shoreline shape. From Figure 4, distribution of lake water surface temperature, which is evenly distributed except on particular area that shows higher temperature, such as on surround the hot water inflow site. Figure 5. shows vertical temperature distribution at A-B cross section. Until the 13th layer or 50 m depth, temperature distribution resulted by model tend close to the observation.

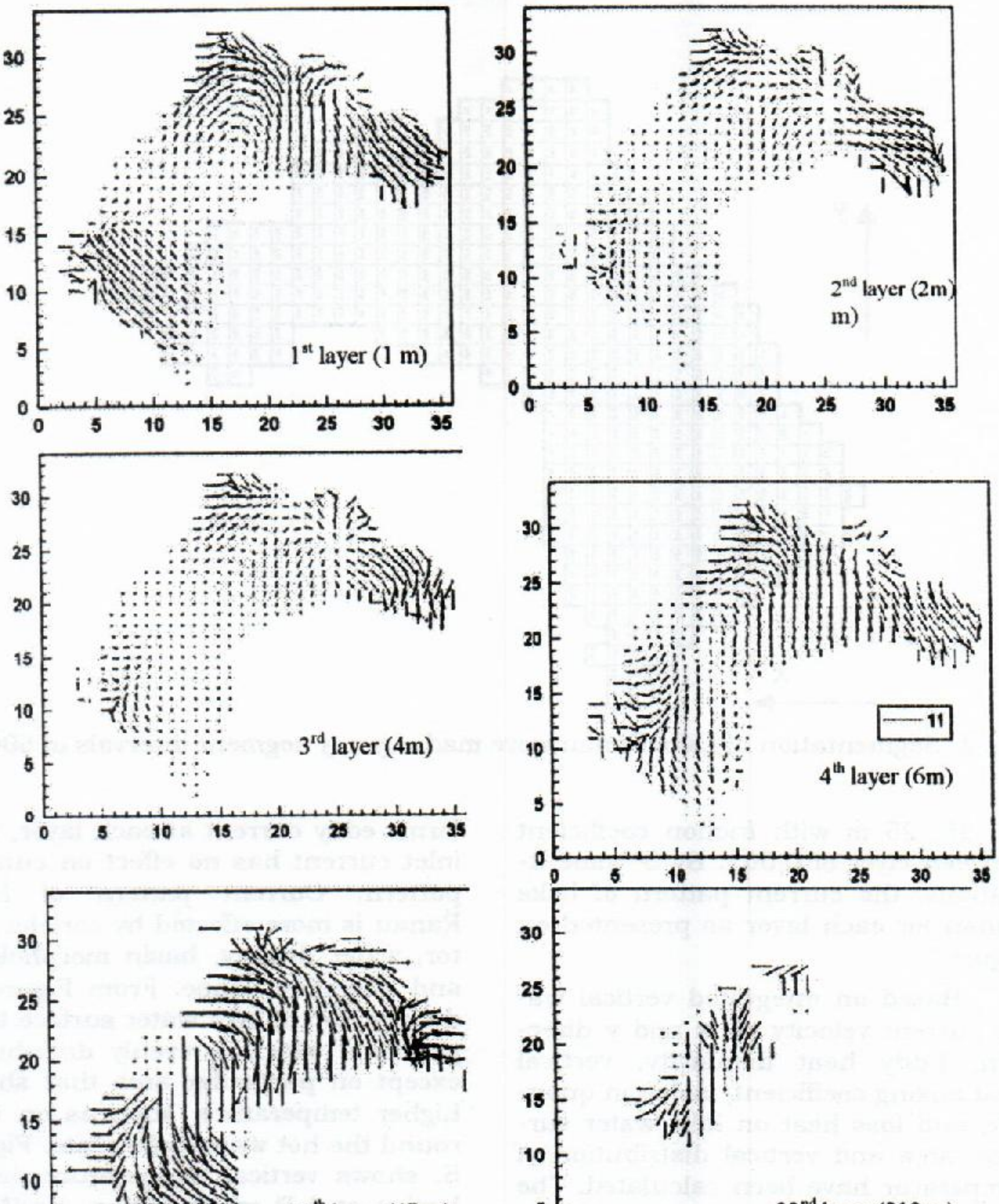

Fig. 3. Current pattern of Lake Ranau.

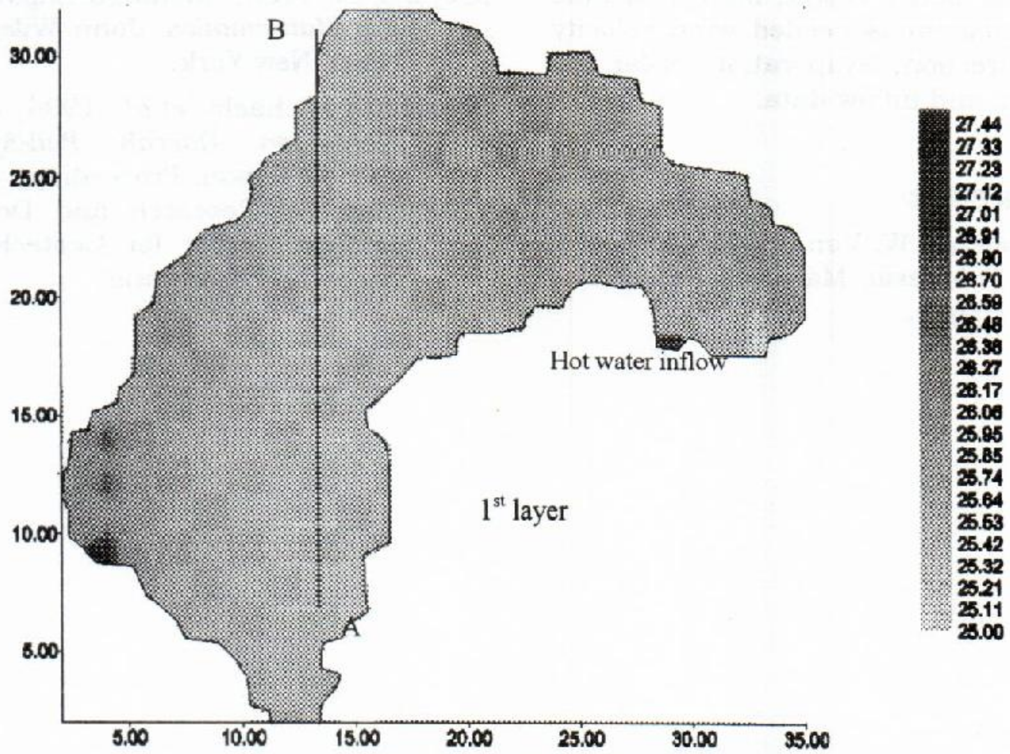

Fig. 4. Area distribution of temperature.

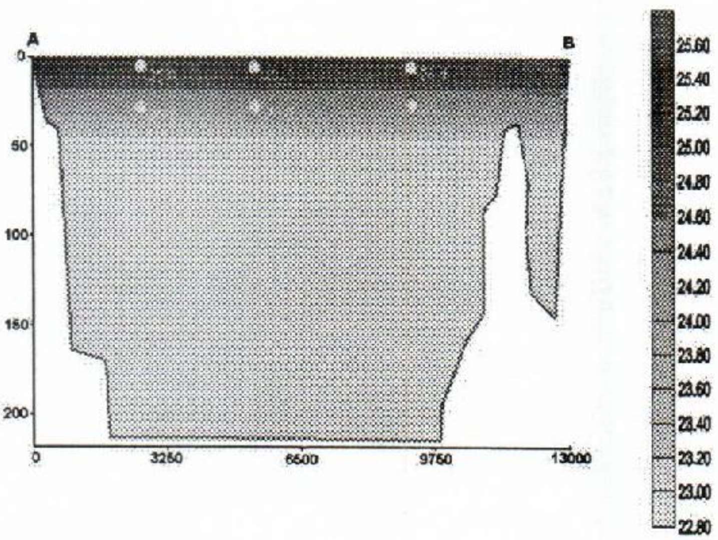

Fig. 5. Vertical distribution of temperature at cross-section A-B.

For better result, more real time measurement is needed wind velocity and direction, evaporation, solar radiation, and inflow data.

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