Out-Of-Sample Diversified Portfolio Performance with Minimum-Variance, Inverse-Variance, And Hierarchical Risk Parity Methods

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ABSTRACT

This research aims to find a portfolio optimization method that has superior performance in out-of-sample simulation in the Indonesia equity market. The portfolio will be optimized utilizing three methods: Inverse-Variance, Minimum-Variance, and Hierarchical Risk Parity. Portfolio performance, i.e. Sharpe Ratio, is calculated using the actual return generated by the portfolio. The accuracy of the portfolio's model is measured using the sum of squared errors and the t-test between the actual return and expected return of the portfolio in out-of-sample simulations. The out-of-sample simulations will be carried out in 3 scenarios of holding period and rebalancing at the end of the holding period. The simulation results suggest that the Hierarchical Risk Parity method outperformed the other two portfolio optimization methods, having the highest actual return, Sharpe ratio, and accuracy in predicting returns. This research concludes that a more sophisticated approach is required to build an equity portfolio with better performance in Indonesia.

Keywords: Portfolio Optimization; Minimum-variance; Inverse-variance; Hierarchical Risk Parity; Out-ofsample; Simulation; Portfolio Performance.

1. INTRODUCTION

In building a portfolio, we are faced with the decision to determine the assets to be included in the portfolio and the allocation of each respective asset. We need to combine the views between expected returns and volatility of the assets to generate the optimal portfolio. This will affect the allocation of each asset when forming an optimal portfolio.

Harry Markowitz first introduced portfolio optimization through his modern portfolio theory that utilizes the Critical Line Algorithm (CLA), a technique for calculating the proportions of each asset in a portfolio. This well-known algorithm guarantees the right solution after several iterations that meet the Karush-Kuhn-Tucker conditions (Kuhn & Tucker, 1952). However, the solution produced by CLA cannot be relied on because of several practical problems. A small deviation from the expected return can generate new portfolio allocations different from the initial allocations (Michaud, 1998). Given that returns can rarely be forecasted with sufficient accuracy, many researchers choose not to rely on the variance-covariance matrix of the portfolio (De Prado, 2016). This has led to the alternative risk-based asset allocation method, e.g. minimum variance and risk parity (Jurczenko, 2015).

De Prado (2016) introduced a new approach in allocating optimal portfolios, i.e. Hierarchical Risk Parity (HRP). The HRP approach can be used for portfolio allocation based on a singular covariance matrix that cannot be estimated by quadratic optimization, which will produce a more stable portfolio compared to a risk-based asset allocation portfolio (De Prado, 2016). HRP approach can be a new option in building an optimal portfolio because it has the capability to deliver superior performance compared to other risk-based asset allocation approaches.

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2. LITERATURE REVIEW

A portfolio management framework begins with stock selection and is followed by the allocation of each stock in the portfolio. This section covers the literature review of the stock selection and portfolio optimization approaches used in this study.

2.1 Single Index Model

Markowitz's modern portfolio theory provides analytical tools for the analysis and selection of the optimal portfolio. In continuance of it, William Sharpe (1968) simplified the amount and type of input data required to perform portfolio analysis. This method is known as the Single Index Model, which uses a cutoff rate, unsystematic risk, and excess return of beta ratio in constructing the optimal portfolio.

In a single index model, the risk of a portfolio depends on the sensitivity of security associated with the movement of the portfolio market return. The single index model, also known as the market model, gave us an estimate of a security's return as well as of the value of the index. This model calculates the correction or variance of each pair of possibility securities in the portfolio. If the total stocks collected in the portfolio increase, the covariance that is calculated will decrease.

The most important advantage of the single index model is the framework it provides for macroeconomic and security analysis in the preparation of the input list that is so critical to the efficiency of the optimal portfolio. Sharpe's single index model is especially useful to construct an optimal portfolio by analyzing how and why securities are included in an optimal portfolio.

The regression equation for the single index model can be written as follows:

$$R_i = \alpha_i + \beta_i R_m + e_i \tag{1}$$

where:

 $R_i =$ expected return on asset i

 α_i = intercept of a straight line or alpha coefficient

 β_i = slope of a straight line or beta coefficient

 R_m = market risk premium

 $e_i = \text{residuals error}$

The number of stocks selected in the portfolio depends on a unique cutoff rate. Stocks with an excess return to the beta ratio $(\underline{R_i} - R_f/\beta_i)$ higher than the cutoff rate are included in the portfolio, while stocks with lower ratio are excluded. The cutoff point is denoted by C*. For the portfolio of i stocks, the calculation for the cutoff rate can be written as follows (Elton et al., 2014):

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^i \frac{\left(\underline{R_j} - R_f\right) \beta_j}{\sigma_{ej}^2}}{1 + \sigma_m^2 \sum_{j=1}^i \left(\frac{\beta_j^2}{\sigma_{ej}^2}\right)}$$
(2)

where:

 σ_m^2 = variance in the market index

 σ_j^2 = the variance of a stock's movement unassociated with the movement of the market index; usually referred to as a stock's unsystematic risk

2.2 Risk-Based Portfolio

In a risk-based portfolio, the key is the level of volatility reduction. Because the investor's objective is always to obtain a better performance, the choice of this parameter is crucial. This relationship depends strongly on the level of the market risk premium. A risk-based portfolio is associated with the concept of diversification. Because a single number cannot measure diversification, practitioners consider a different approach.

There has been increased interest in applying "risk control" or "risk-based" techniques in an asset allocation context. Some examples of techniques proposed in academic history are "1/N" or equal-weighting, minimum variance, maximum diversification, volatility weighting, volatility targeting, and equal risk contribution or "risk parity". Apart from this risk-budgeting perspective, a large part of the literature has promoted risk control as a full-fledged investment criterion, suggesting that controlling the risk dimension is sufficient to build a portfolio or an opportunity to reap risk-adjusted outperformance (Jurczenko, 2015).

2.2.1 Minimum Variance (MV)

In the mean-variance framework, the Minimum Variance portfolio is the sole efficient allocation that dispenses with precise and complete return forecast (Haugen & Baker, 1991). Since the largest source of uncertainty and error comes from estimating return, MV has proven robust in practice (see Clarke, De Silva, and Thorley, 2006). However, MV implicitly assumes that all expected returns are equals. When all investments have the same expected return and are independent of risk, to achieve maximum returns, one should concentrate on minimizing risk only. This is the explicit objective of the minimum variance portfolio. With the Critical Line Algorithm Markowitz, the allocation of a minimum-variance portfolio can be calculated (see Bailey and De Prado, 2013) for a comprehensive implementation CLA). This portfolio still depends on the estimates of the risk and dependencies between assets, with the drawback that errors yielding extreme estimates will also mechanically lead to larger allocation to the badly estimated components, making the optimization an "estimation-error maximization" (Michaud, 1989).

The Minimum Variance Portfolio (MVP) favors low volatility assets and low beta assets and hence benefits from the low volatility anomaly. Several studies have documented that MVPs also pick up other anomalies. Clarke et al. (2006) find the MVP has a subset ally higher value (book-to-price) exposure than the market (since value stocks tend to have low volatilities).

2.2.2 Inverse Variance (IV) Risk Parity

Risk-parity establishes the extension to the volatility-parity weighting scheme and has the goal of distributing the total portfolio risk (volatility) equally across the portfolio's constituents, after accounting for pairwise correlations (Jurczenko, 2015). When all investments have similar expected Sharpe ratios, the correlations between assets are homogeneous (or correlations cannot be estimated reliably); hence the optimal portfolio would be weighted in proportion to the inverse of the assets' volatility. Stocks with higher volatility will get a smaller allocation, and stocks with lower volatility will get a greater allocation. The weights for the inverse variance portfolios can be written as:

$$w^{IVar} = \frac{1/\sigma^2}{\sum_{i=1}^n \frac{1}{\sigma^2}}$$
 (3)

where σ is the vector of asset volatilities and σ^2 is the vector of asset variance.

2.2.2 Hierarchical Risk Parity (HRP)

HRP is a new approach to building an optimal portfolio, introduced by Marco López de Prado in his paper in 2016. The HRP algorithm calculates the inverse-variance weight for groups of similar assets,



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iteratively applied to smaller sub-groups until each asset forms its clusters. According to De Prado (2016), HRP addresses three major concerns of quadratic optimizers in general and Markowitz's CLA in particular: instability, concentration, and underperformance. The three stages of the HRP algorithm in building an optimal portfolio are as follows (see De Prado [2016] for comprehensive steps):

- 1. *Tree clustering* combines assets into a hierarchical structure of clusters so that allocations can flow downstream through a tree graph.
- 2. *Quasi-diagonalization* reorganizes the rows and columns of the covariance matrix so that the largest values lie along the diagonal. Similar assets are grouped, dan dissimilar assets are placed far apart. Assets with higher correlations are placed adjacently and close to the matrix diagonal.
- 3. Recursive bisection. The inverse-variance allocation is optimal for a diagonal covariance matrix. It gives an advantage by allocating funds top-down through the tree structure, in which riskier clusters are given less funding.

The objective of hierarchical clustering in the HRP algorithm is to obtain a quasi-diagonal correlation matrix that is amenable to inverse-variance-based allocation techniques. The most efficient (minimizing variance) combination of uncorrelated time series is inverse-variance weighting (De Prado, 2016). Thus, inverse-variance asset allocation is the most appropriate for assets with an approximately diagonal correlation matrix. One of the main advantages of HRP is its ability to solve the stability problem in computing a portfolio on an ill-degenerated or even a singular covariance matrix. Another study shows that hierarchical clustering-based portfolios achieve statistically better risk-adjusted performances than commonly used portfolio optimization (Raffinot, 2017). Lau et al. (2017) apply HRP to different cross-asset universes consisting of many tradable risk premia indices, and HRP is proven to deliver a superior risk-adjusted return. De Prado (2016) shows that HRP achieves lower out-of-sample volatility and higher risk-adjusted return than inverse-variance allocation.

3. RESEARCH METHODOLOGY

This research aims to recommend the best portfolio optimization methods for the Indonesian equity market. The process begins with selecting stocks and then building a diversified portfolio with three alternative methods: Inverse-Variance, Minimum-Variance, and Hierarchical Risk Parity. The performance of each portfolio will be measured based on its actual return and risk in out-of-sample simulation. An analysis of the three portfolios will be carried out to determine the best approach or optimization method that can be used in the Indonesian equity market.

In this research, we are using the weekly closing price of 71 shares listed on the Indonesia Stock Exchange that meet the following criteria: (i) has been in the LQ45 index (to avoid assets that are less traded and do not have a major influence on market movements and in this research will using Single Index Model as stocks selection method), (ii) actively traded, and (iii) has weekly closing prices data from 2009-2019. These 71 shares will be selected based on the single index model with a cut-off rate. Using this approach, we can figure out which of the shares has the best performance to be included in the portfolio.

We then build the Inverse-Variance Portfolio (IV-P), Minimum-Variance Portfolio (MV-P), and Hierarchical Risk Parity Portfolio (HRP-P) using the selected stocks. All portfolios will be built by using weekly closing prices between April 2009 to October 2018. All portfolios will be tested in an out-of-sample simulation within 48 weeks (from November 2018 to October 2019) with three scenarios of holding period and rebalancing:

- Scenario (1), 4-week holding and rebalancing period
- Scenario (2), 12-week holding and rebalancing period
- Scenario (3), 16-week holding and rebalancing period

The selection of scenarios in 4 weeks, 12 weeks, and 16 weeks were based on the number of multiples.

The number 48 is multiple numbers from numbers 4, 12, and 16; therefore the simulation duration in each scenario will be equal.

The portfolio's actual return is calculated using the following formula:

$$R_p = \sum_{i}^{n} W_i(R_i) \tag{4}$$

where,

 R_p : the actual portfolio's return

 W_i : the weight of asset i

 R_i : the actual return of asset i

n : number of assets in the portfolio

Sharpe ratio measures the reward to volatility, i.e. the ratio of risk premium to the total risk of the portfolio (Bodie et al., 2014). This ratio is widely used to measure the performance of a portfolio and as a comparison measure between portfolios. The formula used to calculate the portfolio's Sharpe ratio is:

Sharpe Ratio =
$$\frac{R_p - R_f}{\sigma_p}$$
 (5) where,

 R_p : the actual return of the portfolio

 R_f : risk-free rate, Indonesian 1-year government bond yield at the evaluation date

 σ_p : portfolio's standard deviation

The accuracy of the optimization method in predicting the portfolio's return will be analyzed using the sum of squared errors (SSE) and t-test. SSE represents the deviation of the actual return from the expected return in the out-of-sample simulation. While the t-test measures whether the average of the actual returns is statistically indifferent from the average of the expected returns.

4. RESULTS

4.1 Stocks Selection

Based on a single index model, 18 stocks had a risk-adjusted return – excess return to beta – higher than the cut-off rate. These 18 shares are expected to provide a higher return than their expected return based on the single index model. Hence, the portfolios will be built using the 18 selected stocks and the three alternative optimization approaches. The optimal portfolio will be used as a portfolio at week-0 in the out-of-sample simulation which will be held and rebalanced according to the predetermined scenario.

4.2 Portfolio Optimization Results

The portfolio optimization with the Inverse-Variance approach yields one stock with a weight of more than 10%, while the lowest weight in the portfolio is 2.25%. In the inverse-variance approach, the weighting process is based on the volatility of each asset. Assets with the largest volatility will get the smallest portion, and assets with the lowest volatility will get the biggest portion of the weight. Therefore, there will be no asset with 0% weight.

Four stocks have 0% weight in the MV-P. The weighting process of the minimum-variance approach focuses on building a portfolio with the lowest standard deviation (global minimum variance). Among



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the three approaches, the minimum-variance portfolio has the slightest standard deviation.

The optimal portfolio built with the HRP approach returns two stocks with a weight of more than 10% and not a single stock with a 0% weight. The matrix diagonalization in this approach is proven to distribute the weight of shares based on the inverse-variance allocation, so there are no shares with a 0% weight.

Table 1. Portfolio Optimization 18 Selected Shares with 3 Approaches (Week-0)

	Expected	Variance	Weight		
Stocks	Return		IV-P	MV-P	HRP-P
CMNP	0.33%	0.36%	5.35%	9.03%	7.18%
GGRM	0.62%	0.24%	8.14%	3.18%	5.30%
HMSP	0.52%	0.14%	13.44%	23.20%	11.30%
INKP	0.70%	0.44%	4.35%	0%	3.23%
KAEF	0.84%	0.51%	3.77%	0%	3.84%
PNIN	0.49%	0.30%	6.52%	7.08%	6.50%
ACES	0.73%	0.28%	6.90%	8.03%	7.13%
BRPT	0.61%	0.57%	3.36%	0.23%	2.50%
BTPN	0.64%	0.25%	7.75%	11.59%	10.47%
CPIN	1.07%	0.48%	4.02%	0%	3.39%
JIHD	0.37%	0.45%	4.33%	6.27%	5.82%
INAF	1.23%	0.86%	2.25%	2.53%	2.33%
MYOR	0.97%	0.26%	7.39%	7.00%	6.23%
NISP	0.33%	0.34%	5.74%	10.95%	7.34%
MAPI	0.87%	0.38%	5.02%	0.48%	3.27%
MNCN	0.63%	0.58%	3.31%	0%	4.23%
TKIM	0.74%	0.47%	4.11%	5.34%	4.19%
TPIA	0.69%	0.45%	4.25%	5.08%	5.75%

4.3 Out-of-Sample Simulation

4.3.1 Portfolio's Actual Return

From the optimization in out-of-sample simulation, the IV-P produces the highest average of expected return and average standard deviation in every scenario. While the MV-P produces the lowest average expected return and standard deviation in every scenario. For the performance based on actual return, Portfolios built using the HRP approach produce the highest average of actual returns for each scenario. The MV-P yields the worst performance of actual return.

Table 2. Comparison of Average Expected Return Portfolio

Caanania	Average Expected Return			Note
Scenario	IV-P	MV-P	HRP-P	
4 weeks	2.28%	2.03%	2.22%	value in 4-weekly
12 weeks	7.05%	6.27%	6.85%	value in 12-weekly
16 weeks	9.47%	6.34%	9.16%	value in 16-weekly

Table 3. Comparison of Average Standard Deviation Portfolio

	Average Standard Deviation			Note
Scenario	IV-P	MV-P	HRP-P	
4 weeks	4.41%	4.01%	4.34%	value in 4-weekly
12 weeks	7.71%	7.01%	7.58%	value in 12-weekly
16 weeks	8.92%	8.09%	8.74%	value in 16-weekly

Table 4. Comparison of Portfolio Performance Based on Average Actual Returns

C	Average Actual Return			Note
Scenario	IV-P	MV-P	HRP-P	
4 weeks	0.43%	-0.07%	0.89%	value in 4-weekly
12 weeks	0.77%	-0.91%	2.13%	value in 12-weekly
16 weeks	0.93%	-0.43%	2.73%	value in 16-weekly

4.3.2 Portfolio's Sharpe Ratio

HRP-P produces a positive and the highest Sharpe ratio for every scenario in the out-of-sample simulation. At the same time, MV-P has the worst-performing portfolio among the three portfolios based on the Sharpe ratio.

Table 5. Comparison of Portfolio Sharpe Ratio

Scenario	Sharpe Ratio			Note
	IV-P	MV-P	HRP-P	
4 weeks	1.03%	-11.33%	11.63%	value in 4-weekly
12 weeks	-4.98%	-29.47%	12.87%	value in 12-weekly
16 weeks	-6.82%	-24.33%	13.63%	value in 16-weekly

4.3.3 SSE and t-test

In the 4-week holding period scenario, the MV-P presents the lowest SSE among the three portfolios. However, the HRP portfolios generate the lowest SSE in 12-week and 16-week holding period scenarios. All portfolios produce a significant t-test result; hence there is no significant difference between the expected return and the actual return of the portfolio built using all three approaches.

Table 6. Comparison of Portfolio Sum of Squared Errors

C	Sum of Squared Errors				
Scenario	IV-P	MV-P	HRP-P		
4 weeks	2.13%	3.24%	2.8%		
12 weeks	2.88%	3.24%	2.0%		
16 weeks	3.55%	4.33%	3.0%		



Table 7. Portfolios T-Test Result (\alpha = 0.05)

Scenario	Portfolio	IV	MV	HRP
	T stat	1.3342	3.1715	0.7238
41	T crit	4.3009	4.3009	4.3009
4 weeks	P-value	0.2604	0.0887	0.4041
	Significant	Yes	Yes	Yes
	T stat	2.1536	5.8203	1.8257
12 weeks	T crit	5.9874	5.9874	5.9874
12 weeks	P-value	0.1926	0.0524	0.2254
	Significant	Yes	Yes	Yes
	T stat	1.4591	2.9860	1.0175
16 weeks	T crit	7.7086	7.7086	7.7086
	P-value	0.2936	0.1591	0.3702
	Significant	Yes	Yes	Yes

5. DISCUSSION

In the out-of-sample simulations (using the same stocks), the HRP method generates the best actual return among the three approaches. The HRP method successfully distributes the weighting of stocks effectively. With 18 shares selected from the selection process, the HRP method forms 16 clusters based on the proximity of the Euclidean distance. The weighting based on inverse variance on these clusters effectively delivers superior performance in out-of-sample simulation. The three steps in the HRP method: tree clustering, matrix diagonalization, and recursive bisection, are proven to be able to build an optimal portfolio for the Indonesia equity market. In a certain period – in the out-of-sample simulation – the HRP portfolio produced a positive actual return while MV and IV portfolio generated a negative actual return. The HRP portfolio provides a positive Sharpe ratio in every scenario having larger average actual returns than the risk-free rate.

All portfolios have significant t-test results, i.e. there is no significant difference between the expected and the actual returns. The HRP portfolio has the most significant p-value among the three approaches, hence providing more confidence. Based on SSE, the MV portfolio resulted in the smallest SSE in the 4-week holding period scenario, but the HRP portfolio generated the smallest SSE in 12-week and 16-week holding period scenarios. Based on this result, the HRP portfolio is more accurate in predicting actual returns for a holding period of more than 12 weeks because HRP outperforms the other two portfolios in a 12-week and 16-week holding period based on SSE and t-test.

6. CONCLUSION

Optimization of the HRP portfolio did not result in the highest expected return and lowest standard deviation compared to the IV and MV portfolios, yet the HRP portfolio was able to outperform the other portfolio based on actual return and Sharpe ratio in the out-of-sample simulation. Furthermore, the other factor that supports the outperformance of the HRP portfolio compared to the other two approaches is the accuracy of the HRP portfolio based on SSE and p-value in the out-of-sample simulation.

This result is identical to the previous research by De Prado (2016) using a Monte Carlo simulation that proves the HRP portfolio can outperform the minimum-variance and inverse-variance portfolios. Investors or investment managers who want to build a portfolio of stocks in the Indonesian equity market are recommended to use the HRP approach. The high level of private information in the Indonesian market (Arief & Husodo, 2019) suggests the need to apply a more sophisticated or complex approach in building a portfolio to achieve optimal performance.

This research is limited by the absence of transaction costs and the number of shares included in the portfolios in the out-of-sample simulation. The transaction cost and limitation on the purchasing number of shares may lead to a different result in the portfolio's actual return. Adding more assets into the portfolio without considering the aspects of the assets (adding assets randomly) will not guarantee to improve the portfolio's performance. Selecting assets based on the similarity aspect (such as the Sharpe Ratio or Treynor Ratio) can be considered in future studies to see whether using a larger sample will affect the performance of these portfolios. For further research, stock selection based on higher-frequency liquidity proxies can be applied in building an optimal portfolio to provide better returns by utilizing the potential of private information on the Indonesian market. Several other methods may be applied in a stock selection process, e.g. based on liquidity proxies in higher frequencies data (percent-cost proxy, LOT mixed impact, High-Low impact, and FHT impact [Fong et al., 2017]).

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