

VIRTUAL POPULATION ANALYSIS AND COHORT ANALYSIS IN FISHERY STOCK ASSESSMENT

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ABSTRAK

"VIRTUAL POPULATION ANALYSIS" DAN "COHORT ANALYSIS" UNTUK PENDUGAAN UKURAN SEDIAAN SUMBERDAYA PERIKANAN. *Pengkajian tentang sumberdaya perikanan, baik yang menggunakan "surplus production model" maupun "dynamic pool model", mendasarkan kepada anggapan bahwa hasil tangkapan per satuan upaya penangkapan (catch per unit effort, CPUE) dapat digunakan sebagai indeks dari kelimpahan nisbi. Namun sayang, bahwa upaya penangkapan (fishing effort) jarang dapat ditera dengan tepat, terutama bila berhadapan dengan perubahan-perubahan yang pesat di bidang teknologi penangkapan, sifat perikanan itu sendiri, serta sasaran atau target utama dari usaha suatu penangkapan.*

GULLAND (1965) mengetengahkan suatu model yang lazim dikenal dengan nama "virtual population analysis (VPA)" untuk menduga ukuran sediaan serta laju kematian akibat penangkapan untuk masing-masing kelompok umur dari suatu populasi ikan. Meskipun cara ini bebas dari penggunaan CPUE berikut beberapa kelemahan yang terkait dengannya, namun ini tidak berarti bahwa VPA juga luput dari berbagai keterbatasan yang selanjutnya akan diuraikan di dalam tulisan ini.

POPE (1977) mengemukakan model yang ia namakan "cohort analysis" yang merupakan versi lain dari VPA, dan ternyata penggunaan model ini untuk maksud yang sama, jauh lebih sederhana dibanding dengan penggunaan VPA.

Lebih lanjut JONES (1974, 1979, 1984) dan PAULY (1984) mencoba menerapkan kedua model tersebut, tetapi tidak berdasarkan data kelompok umur, melainkan berdasarkan data kelompok panjang badan ikan. Bagi perikanan di Indonesia, serta perairan tropis lainnya, di mana cara penentuan umur ikan masih belum terpecahkan sepenuhnya, model yang terakhir ini nampak memberikan harapan untuk dikembangkan lebih lanjut.

INTRODUCTION

Traditionally, fishery management were based primarily upon the theory of surplus production models of GRAHAM and SCHAFER developed in the 1930's and 1950's as well as upon the analytical models of RICKER and BEVERTON and HOLT developed in the 1940's and 1950's. These advances in the fishery science were base on the assumption that catch per unit effort (CPUE) could be used as index of relative abundance in analyzing of exploited fish populations. Unfortunately, the interpreta-

tion of CPUE data becomes further complicated since rapid changing in fishing technology and increasing in fishing effort, changing nature of fisheries, and variability of the target species, will make it difficult to estimate fishing effort precisely

In multispecies fisheries, separation of directed effort from total effort will introduce ambiguities into the estimate of effort, so that consistent CPUE estimates will be difficult to maintain. As a result of these problem the analysis of exploited

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fish stocks which not reliance on CPUE were then developed.

GULLAND (1965) provided virtual population analysis (VPA) for estimating age specific fishing, mortality rate and cohort size from catch at age data, with natural mortality rate assumed constant with age and known terminal fishing mortality rate.

POPE (1972) devised an approximate version of VPA, generally called as cohort analysis, which is computationally very much simpler to carry out.

JONES cohort analysis (1974, 1979, 1984) or length cohort analysis is a modified techniques of conventional cohort analysis of Pope (1972) which based on catch at age data to be applicable to catch at length data which represent the catch from fisheries of steady state conditions. PAULY (1984) developed length structured VPA model analogous to JONES length cohort analysis.

VITUAL POPULATION ANALYSIS (VPA)

In general, virtual population analysis is a method for estimating population size given estimate of terminal fishing mortality, catch at age data and natural mortality rate. The term of "virtual population" was introduced by FRY (1949) as the sum of all individual fish belong to a given year-class present in the population at any given time, that would ultimately appear in the catch in that year and in all subsequent years.

This method assumes that relatively large part of the total removals are due to fishing that aged without error and all removals are account for in the catch except those of natural losses. Additionally, it assumes that all fish become available to the fishery at some time in their life, with constant natural mortality rate.

VPA is an iterative procedure for extracting estimates of cohort size and fishing mortality which relies on two equations

commonly encountered in fisheries population dynamics; the first is the catch equation of BARANOV (BEVERTON & HOLT 1975) which expresses catch rate in number instead of in weight as :

$$aC_y = (aF_y/aZ_y) (1-\exp(-aZ_y)) aN_y \dots (1)$$

- where y = year (y = 1,2, Y)
- a = age of cohort (a = 1, 2, A)
- aC_y = catch in number of age a in year y
- aN_y = number of age a fish in the beginning of year y
- aZ_y = total mortality rate on age a in year y
- aF_y = fishing mortality rate on age a in year y

$$(aZ_y = M + aF_y)$$

M = natural mortality rate.

In other words, Eq. (i) can be described that the catch from a population of fish during a unit time period is simply equals to the fraction of the deaths due to fishing multiplied by the fraction of the total deaths and the population size existing at the beginning of the time period.

The second equation is based upon the assumption that within any one age group the survival model can be represented by exponential curve with a constant mortality rate, that is:

$$a+1N_y+1 = aN_y \exp(-aF_y + M) \dots (2)$$

Equation (1) and (2) can be combined together to give

$$\frac{a+1N_y+1}{aC_y} = \frac{(aF_y + M)\exp(-aF_y + M)}{aF_y (1-\exp(-aF_y + M))} \dots (3)$$

Parameter Estimate Procedure

Para meters are estimated separately for each year. Given for aC_y, aF_y (terminal fishing mortality rate where a = A and y = Y), and M, then Eq. (1) and (2) are solved iteratively in a backwards or hind cast mode

for aN_y and aF_y for all past years of life of the cohort in the following way. Using an estimate of terminal fishing mortality aF_y and observed catch aC_y ($a = A$), Eq. (1) is solved for aN_y ($a = A$), either from $aN_y = aC_y \cdot aZ_y / (aF_y(1 - \exp(-aZ_y))) \dots(4)$

if aC_y and aF_y ($a = A$, and $y = Y$) represent the estimated terminal catch and fishing mortality for the age $a = A$ only, or from

$$aN_y = aC_y \cdot aZ_y / aF_y \dots (5)$$

if the estimated for aC_y and aF_y represent the catch and fishing mortality for the age $a = A$ and all subsequent years. In this case, the oldest age group refers to all fish of age $a = A$ and older. Theoretically, the time regarding with age infinity is equal to ∞/∞ , consequently $t = \infty/\infty$ and therefore

$$\exp(-aZ_y \cdot t) = \exp(-aZ_y \cdot \infty/\infty) = \exp(-\infty/\infty) = 0$$

thus

$$aN_y = (aC_y \cdot aZ_y) / (aF_y(1 - \exp(-aZ_y \cdot t)))$$

$$= aC_y \cdot aZ_y / aF_y$$

(as expressed in Eq. (5)).

The next step is using Eq. (3) to solve for aN_{y-1} , and finally Eq. (2) can be used to estimate aN_{y-1} and so on until the youngest cohort is done. RICKER (1948) described the mechanics of these sequential computations of these two Equations and this method then was popularized by GULLAND (1965).

Numerical Example of Virtual Population Analysis

Input data

Basic input data for VPA and its version methods consist of estimates of catch in numbers at success ice age (aC_y) which can be the numbers of catch-at-age for a single cohort followed through its life, or alternatively, these data can be representative of the numbers of catch-at-age from a population if it were exploited under steady-state or equilibrium conditions.

Other input data contain an estimate of instantaneous natural mortality rate M , and a value of terminal instantaneous fishing mortality rate aF_y , i.e. the fishing mortality rate for the oldest age group ($a = A$).

Steps in the computations

A numerical example of virtual population analysis (VPA) is given in Table 1.

Table 1. Virtual population analysis (VPA) with oldest age consisting of fish age 6 years and older.

Age group	aC_y (millions)	aN_y (millions)	aF_y
0	599	4 306.6	0.17
1	860	2 974.7	0.38
2	1 071	1 665.5	1.12
3	269	444.9	1.07
4	69	125.0	0.93
5	25	40.4	1.10
6 & older	8	11.2	0.50

(Modified from Table 5.1.1. page 215 in SPARRE 1985).

Input data : $M = 0.2$ and $F = 0.50$.

The first step is to determine the abundance of fish age 6 years and older, assumed that $M = 0.2$ and terminal $F = 0.5$, by employing Eq. (5)

$$6N = 6C(6F + M) / 6F$$

so that

$$6N = 8(0.5+0.2)/0.5 = 11.2$$

The second step proceeds by using Eq. (3), which for estimating fishing mortality rate of 5-year age group we have the relationship of:

$$11.2/25 = (5F + 0.2) \exp(-5F + 0.2) / 5F (1-\exp(-5F+0.2))$$

We have obtained an equation with $5F$ is the only unknown variable. Since the equation above cannot be solved by algebraic calculation, then it should be solved by trial and error method until the left hand side equals to the right hand side, and we have got $5F = 1.10$.

The third step is employing Eq. (2), i.e. by using exponential survival model we are able to estimate, the abundance of the subsequent year :

$$5N = 11.2 \exp(-1.10 + 0.2) = 40.4$$

The next abundance and fishing mortality can be calculated exactly similar to those for the above calculations in a backwards way.

COHORT ANALYSIS

Instead of iteratively solving VPA of GULLAND's method (1965), POPE (1972) simplified the procedure somewhat by introducing a discrete approximation to the continuous exponential survival model of Eq. (2). He assumed that the whole of the catch is taken at exactly the middle of the age interval and that only natural mortality rate takes place continuously on an exponential basis (Figure 1).

At the middle of the age interval, just after the catch has been taken from the fishery, the number of survivors is given by :

$$a+1N_{y+1} \exp (M/2) \text{ (i.e. at point C)}$$

also at the middle of the age interval, just before the catch is taken, the number of individuals is given by :

$$a+1N_{y+1} \exp (M/2) + aC_y \text{ (i.e. at point B)}$$

where aC_y is the catch during age a and $a+1$ interval.

The estimated of aN_y (i.e. point A) can be approximated by proceeding backwards one more step to the beginning of the interval, which can be defined by the equation below :

$$aN_y = (a+1N_{y+1} \exp (M/2) + aC_y) \exp (M/2) \dots (6)$$

POPE (1972) demonstrated that within the range of $M < 0.3$ and $F < 1.2$, the error of the approximation will less than 4 per cent). Numerical analysis proceeds by first, determining the number of individuals for the oldest age, i.e. aN_y which $a = A$ and $y = Y$, and then $a-1F_{y-1}$ is obtained directly from

$$a-1F_{y-1} = \ln (a-1N_{y-1}/aN_y) - M \dots (7)$$

where $\ln (a-1N_{y-1} / aN_y)$ is equal to Z , total mortality rate.

Numerical Example of Cohort Analysis

Input data

Basic input data for the computation of cohort analysis are similar to those used in VPA, which consist of estimates of numbers caught at successive ages (aC_y), an estimate of natural mortality rate M , and a value of F , i.e. the instantaneous fishing mortality rate of the terminal age group.

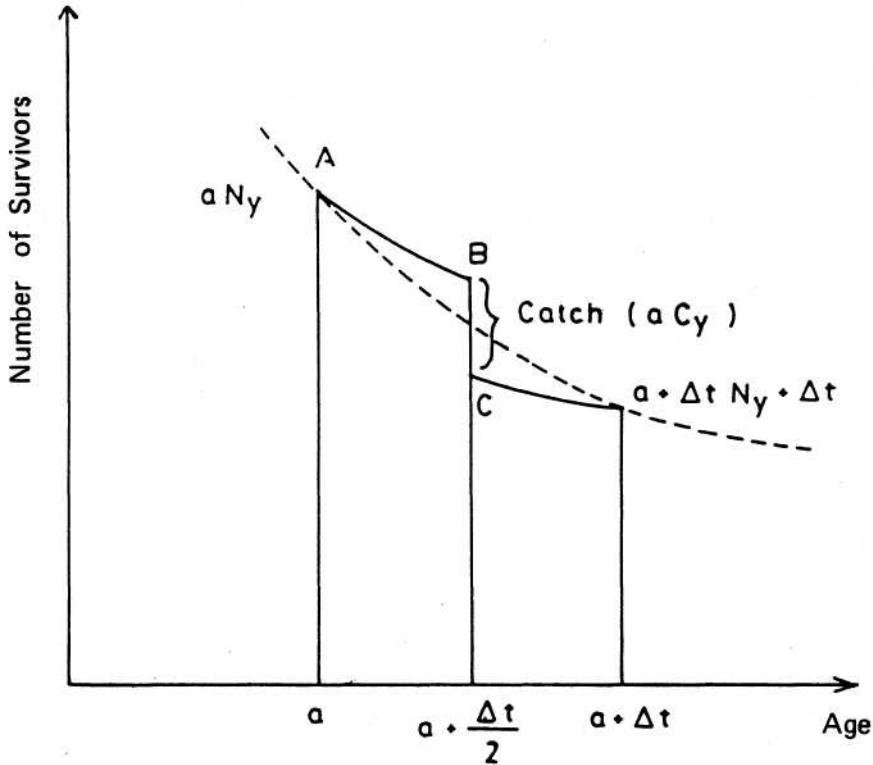


Figure 1. Illustration of the Pope's cohort analysis, an exponential curve is replaced by a discrete approximation function.

Steps in the computations

Let us the catch-at-age data of Table 1 for the example of cohort analysis computations, then put the complete computations in Table 2.

The computation proceeds by starting with the terminal oldest age group and proceeding backwards until the youngest. To determine ${}_6N$ and older, i.e. the number of individuals that reaching age 6 years and older, we employ Eq. (5) :

$${}_6N = 8 (0.2 + 0.50) / 0.50 = 11.2$$

Further computation proceeds by using Eq. (6), that is:

$${}_5N = (11.2 \exp (0.2/2) + 25) \exp (0.2/2) = 41.3$$

The next abundance can be calculated similarly to this of the above calculation in a backwards mode.

Calculation of the instantaneous fishing mortality rate per each group can be directly performed from :

$${}_5F = \ln(41.3/11.2) - 0.2 = 1.11$$

Proceeding in this way, value of aF_y for each age group can be determined.

Alternative input data for oldest age group

If aC_y and aF_y represent the catch and fishing mortality rate for the oldest age group only, then we must employ Eq. (4) to estimate the cohort size, i.e.

$$aN_y = aC_y, aZ_y / (aF_y(1 - \exp(-aZ_y)))$$

Table 2. Cohort analysis with oldest age group consisting of fish age 6 and older.

Age group	aCy (millions)	aNy (millions)	aFy
0	599	4 412.9	0.17
1	860	3 070.8	0.37
2	1 071	1 735.9	1.14
3	269	452.1	1.07
4	69	126.7	0.92
5	25	41.3	1.11
6 & older	8	11.2	0.50

(Modified from Table 5.1.1. page 215 in SPARRE 1985).

Input data : $M = 0.2$ and F terminal = 0.50.

It is worthy to note that in this calculation nothing needs to be known about the history of the older age groups, as a result, this would be an appropriate procedure for dealing with a migratory species for which the older age groups may not urgently be present in the fishery.

LENGTH (JONES) COHORT ANALYSIS

For any time interval (Δt), the simplified model of POPE (1972), i.e. Eq. (6) can be generalized as:

$$aNy = (a + \Delta t Ny + \Delta t \exp(M\Delta t/2) + aCy) \exp(M\Delta t/2) \dots (8)$$

where Δt is the time required to grow from the beginning to the end of a length interval, and with all the other parameters defined as in the Eq. (1). In this form, the equation allows for application to the catch-at-length data, if and only if Δt can be defined.

There are various methods of allowing for growth, the simplest and widely used in fishery population dynamics is the VON BERTALANFFY growth model (1938), i.e.:

$$L_t = L_{\infty} (1 - \exp(-k(t - t_0)))$$

where L_{∞} is asymptotic length, k is growth coefficient, and t_0 is theoretical time when length is equal to zero. This conventional growth curve expresses length as a function of age, i.e. corresponding to each age there is a mean length, consequently L_{∞} may be less than the largest individual (Figure 2).

Rearrangement of the von Bertalanffy growth equation gives an expression for age as a function of length.:

$$t = t_0 - (1/k) \ln(L_{\infty} - L_t / L_{\infty})$$

i.e. corresponding to each length there is a mean age, so that the value of L_{∞} will never be smaller than the largest individual. Jones cohort analysis requires the second relationship.

This inverse von Bertalanffy growth model can be used to determine the time required to grow from the lower limit of a length interval (L_1) to its upper limit (L_2). if t_1 and t_2 regarding to the age of L_1 and L_2 , then we have got :

$$t_1 = t_0 - (1/k) \ln(1 - L_1 / L_{\infty})$$

$$t_2 = t_0 - (1/k) \ln(1 - L_2 / L_{\infty})$$

Then Δt , i.e. time required to grow from L_1 to L_2 can be determined as :

$$\Delta t = t_1 - t_2 = (1/k) \ln (L_{\infty} - L_1) / (L_{\infty} - L_2) \dots (9)$$

i.e., a function of k and L_{∞} which independent to t_0 . If the values of L_{∞} and k are known, it is possible to use Eq. (9) to calculate the value of Δt for each length group. In principle, length cohort analysis can then be carried out exactly by employing Eq. (8).

Converting the generalized cohort analysis of Eq. (8) into a version based upon catch-at-length data, we need the value of $\exp (M\Delta t/2)$. From Eq. (9), the value of $M\Delta t/2$ can be defined as :

$$(M/2) \Delta t = (M/2k) \ln (L_{\infty} - L_1) / (L_{\infty} - L_2)$$

so that

$$\begin{aligned} \exp (M\Delta t/2) &= \exp ((M/2k) \ln ((L_{\infty} - L_1) / (L_{\infty} - L_2))) \\ &= \exp (\ln ((L_{\infty} - L_1) / (L_{\infty} - L_2)))^{M/2k} \\ &= ((L_{\infty} - L_1) / (L_{\infty} - L_2))^{M/2k} \end{aligned}$$

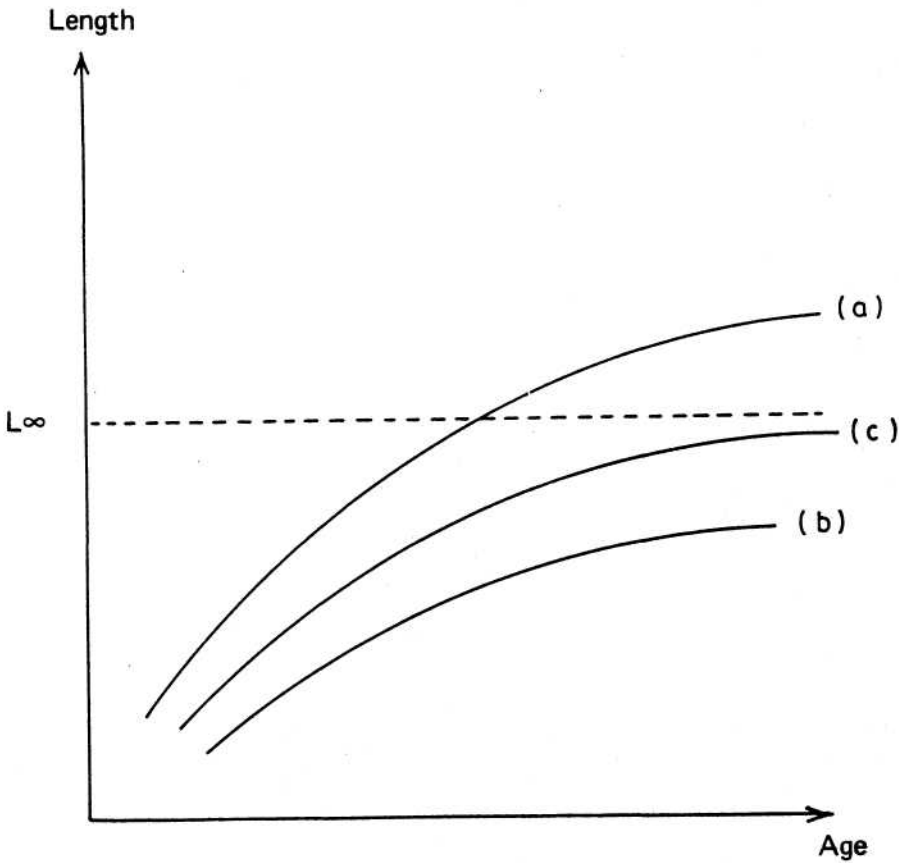


Figure 2. Growth curves for fast growing (a) and slow growing (b) fish, and the relation between mean length and age (c). L_{∞} may be less than the largest individual.

Finally, Eq. (8) can be expressed as:

$N(L_1) @ (N(L_2), X(L_1, L_2) + C(L_1, L_2))$
 $X(L_1, L_2) \dots$ (10) where $N(L_1)$ and $N(L_2)$
 are the number of individuals which reach
 length L_1 and L_2 ; $C(L_1, L_2)$ is the numbers of
 individuals caught of length between L_1
 and L_2 , and

$$X(L_1, L_2) = ((L_{00} - L_1)/(L_{00} - L_2))$$

$$M/2k \dots$$
 (11)

To employ Eq. (10) only two variables are
 required, i.e., L_{00} and M/k in which a
 wide range of values of M/k has been tabulated
 by BEVERTON & HOLT (1966), while Eq.
 (8) although it will lead to identical of
 numerical results it requires more variables, i.e.
 L_{00} , M , and K . Consequently, Eq. (8) is not
 so commonly applicable in practice as to Eq.
 (10)

Procedure of Fitting Length Cohort Analysis

Table 3 shows a worked example of
 cohort analysis applied to a length based

version, which input data represent the
 length composition of the catch.

Initial input data

Input data includes a value of L_{00}
 (let equal to 80 cm), a value of natural
 mortality rate $M = 0.20$, and a value of $k =$
 0.2 . In the length based cohort analysis, a
 value of the rate of exploitation (F/Z) is
 needed rather than the terminal fishing mortality
 rate, let $F/Z = 0.5$.

Steps in the computations

The first step is estimating the value for
 the number of fish reaching to the length of
 70 cm and larger. Allowing for the given catch
 of 0.25 millions of fish at 70 cm length and
 larger, the abundance of fish belongs to this
 length group can be determined by employing
 Eq. (5), we get

$$N(70 \text{ and larger}) = 0.25 / 0.5 = 0.5 \times 10^6$$

The next step of the computation is to de-
 termine $X(L_1, L_2)$ of Eq. (11). For ex-
 ample, for the 65-70 length group.

Table 3. Length cohort analysis based on catch-at-length data.

Length Group	aCy (millions)	$X(L_1, L_2)$	aNy (millions)	aFy
20 – 25	0.10	1.044	50.48	0.005
25 – 30	0.47	1.049	46.18	0.023
30 – 35	3.88	1.054	41.53	0.196
35 – 40	5.54	1.061	33.70	0.323
40 – 45	5.37	1.069	24.73	0.391
45 – 50	4.62	1.080	16.62	0.455
50 – 55	3.03	1.095	9.97	0.434
55 – 60	1.68	1.118	5.54	0.361
60 – 65	1.02	1.115	2.93	0.243
65 – 70	0.46	1.225	1.31	0.260
70 & larger	0.25		0.50	

(Modified from Table 5, page 93 in JONES 1984)

$$X(65,70) = ((80 - 65) / (80 - 70))^{0.5} = 1.225$$

Values of all $X(L_1, L_2)$ are calculated in the same way for all length groups.

Employing Eq. (10) gives $N(65)$, the individual reaching the length 65 cm :
 $N(65) = (0.5 \times 1.225 + 0.46) 1.225 = 1.31 \times 10^6$

In the same manner, the stock numbers reaching each length are estimated by backwards calculations for the successively smaller length groups.

Fishing mortality rate for each length group can be estimated either by employing

$$aFy = \ln(aN_y/a_{+1}N_{y+1}) - M$$

or, more conveniently by

$$F = M(F/Z) / (1 - (F/Z))$$

where the value of exploitation rate F/Z is

$$F/Z = (\text{number caught}) / (\text{number dying})$$

for example, for the 20–25 cm length group

$$F/Z = 0.10 / (50.48 - 46.18) = 0.023$$

The complete of the computations of the length cohort analysis is given in Table 3.

LENGTH – STRUCTURED VPA MODEL

This method is analogous to Jones cohort analysis and reviewed by PAULY (1984). The length-structured VPA model can be derived from generalizing Eq. (3) for any time interval of Δt , becomes

$$a_{+\Delta t}N_{y+\Delta t} / a_{Cy} = a_{Zy} \exp(-a_{Zy}\Delta t) / a_{Fy} (1 - \exp(-a_{Zy}\Delta t)) \dots (12)$$

$$a_{Cy} = a_{+\Delta t}N_{y+\Delta t} (a_{Fy}/a_{Zy}) (\exp(a_{Zy}\Delta t) - 1) \dots (13)$$

Substituting Δt of Eq. (9) into Eq. (12), then Eq. (12) can be used in similar way to Eq. (3) to estimate, first, the terminal fish abundance, and then in a backwards manner, the abundances of the smaller size classes and the fishing mortality rates working on them, in the similar procedure as those employed in (age-structured) virtual population analysis.

CONCLUDING REMARKS

The advantages of the VPA and its version models are that this model is easy to carry out, independent of errors associated with CPUE, no assumptions required regarding to catchability and vulnerability to fish concerned, and it is actually valuable for fishery scientists to understand a fishery in a historical sense which illustrate its population dynamics.

Although VPA and its versions independent of CPUE and its associated errors, they are subject to errors from the assumed values of constant natural mortality rate and the estimated terminal fishing mortality rate as well as estimates of the catch at each age (or at each length) data which must be measured without error. The results also can be biased by the emigration or immigration processes of the defined stock area.

The assumption of the constant natural mortality rate is extremely wrong, since disease or predation vary with age and year contribute to M , besides natural mortality rate of males apparently different from that of females.

The effects of these sources of errors on the results from VPA or cohort analysis have been examined by a number of authors which, in general, can be summarized in Table 4. AGGER *et al.* (1971) investigated the effect of simultaneous errors in the terminal fishing mortality estimate and the natural mortality estimate for simplified VPA model in which natural mortality is constant for exploited ages. POPE (1972) analysed the errors in the fishing mortality and cohort size estimates caused by errors in either terminal fishing mortality rate or catch-at-age estimates. Further, ULLTANG (1977) investigated the effects on VPA estimates of random fluctuations in natural mortality, of seasonal variations in fishing and natural mortality, and of stock migration.

Table 4. Summary of the effects of error on the results of VPA and its versions (HOAG & McNAUGHTON 1978).

Error	Effects of Error	
	Fishing Mortality	Abundance
M Overestimated	Underestimated	Overestimated
M Underestimated	Overestimated	Underestimated
F. Underestimated	Underestimated	Overestimated
F. Overestimated	Overestimated	Underestimated
Immigration	Underestimated	Overestimated
Emigration	Overestimated	Underestimated
aCy Overestimated	Overestimated	Overestimated
aCy Underestimated	Underestimated	Underestimated

Since VPA, cohort analysis, and their versions above apply to only a single cohort of fish, modified equations applied for two or more cohorts simultaneously are developed by DOUBLEDAY (1971), POPE (1977), FOURNIER & ARCHIBALD (1982), and POPE & SHEPHERD (1982). They used least squares approach which allows for examination of the reliability of the parameter values, i.e. cohort size and fishing mortality rate. Although these models may be useful, but in general, they become more complex than VPA or cohort analysis, and especially for DOUBLEDAY's method, it may have some serious imperfections.

HELGASON & GISLASON (1979) and MAJKWOSKI (1981) modified VPA and cohort analysis for simultaneous application to multispecies. They assumed that natural mortality rate M for a given species consists of two components, one component is a function of the abundance of other commercial predatory species and the other component is a constant.

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