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FOX MODEL AND GENERALIZED PRODUCTION MODEL ANOTHER VERSIONS OF SURPLUS PRODUCTION MODELS

by

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ABSTRAK

MODEL DARI FOX DAN "GENERALIZED PRODUCTION MODEL" BENTUK LAIN DARI PADA "SURPLUS PRODUCTION MODELS". *Model dan GrahamSchaefer mendasarkan diri pada beberapa sifat khusus, antara lain bahwa pertumbuhan biomassa mengikuti pola pertumbuhan logistik, juga bahwa penurunan hasil tangkapan per satuan upaya penangkapan (CPUE) terhadap upaya penangkapan (fishing effort) mengikuti pola regresi linier, serta hubungan antara hasil tangkapan (yield) dan biomassa berbentuk parabola yang simetris dengan titik puncaknya (maximum) pada tingkat biomassa sebesar B/2.*

Model dari Fox (1970) memiliki beberapa karakteristik yang berbeda dari model Graham-Schaefer, yaitu bahwa pertumbuhan biomassa mengikuti model pertumbuhan dari Gompertz, dan penurunan CPUE terhadap upaya penangkapan mengikuti pola eksponensial negatif yang memang lebih masuk akal dibandingkan dengan pola regresi linier.

Sedang "generalized production model" dari PELLA & TOMLINSON (1969) sama sekali meninggalkan sifat-sifat khusus yang dimiliki oleh model Graham-Schaefer yang telah disebutkan di atas, ialah bahwa dengan memasukkan sebuah perubah (variable) m *ke dalam model Graham-Schaefer, akan berarti bahwa MSY dapat dihasilkan dari berbagai ukuran biomassa yang bervariasi dari 0 sampai dengan* B_∞. *Untuk menentukan besarnya empat buah parameter dalam model Pella dan Tomlinson tersedia program komputer GENPROD (PELLA & TOMLINSON 1969) yang kemudian di-sempurnakan menjadi GENPROD-2 (ABRAMSON 1971). Penggunaan model Pella dan Tomlinson dalam praktek sehari-hari masih belum terbukti kegunaannya.*

INTRODUCTION

Mathematical models used in the study of fishery population dynamics may be classified into two fundamental approaches. Surplus production models, such as those of GRAHAM (1935) and SCHAEFER (1954, 1957) assume that the rate of population growth subsumes the processes which take place in the fish population (i.e. recruitment, growth, and natural mortality) as a single entity. Dynamic pool models, such as those of BEVERTON & HOLT (1957) try to describe the dynamic of the exploited population in terms of its population parameters, i.e. recruitment, growth, and natural mortality.

Inasmuch, as the surplus yield models require only catch and effort data, these models are particularly useful whenever knowledge of the biological information

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to describe recruitment, growth, and mortality, are insufficient. Conversely, the application of the dynamic pool models requires more detailed biological information about population, which is in general obtained through analysis of catch and effort, age composition, and mark-recapture experiment data.

Graham-Schaefer models assume logistic growth of population and set up two basic result, i.e. (1) fishing effort is a linear function of population size (or catch per unit effort), and (2) yield is a parabolic function of either population size or fishing effort.

Fox model (1970) employes the Gompertz growth function to analyse surplus production model, which result in an exponential relationship between fishing effort and population size, and an asymmetrical yield curve.

A general version of the surplus production model was invented by PELLA & TOMLINSON (1969) which released the restriction Graham-Schaefer model, by assuming that maximum equilibrium yield Y_{max} can be associated with any value of biomass *B,* including $B_{opt} = B_\infty/2$, occuring in the interval from *0* toB_∞.

DESCRIPTION OF THE FOX MODEL

The basic mathematical expression of the Graham-Schaefer models for an exploited fishery population under equilibrium conditions can be formulated as follows:

$$Y_E = kB\left(\frac{B_\infty - B}{B_\infty}\right) \quad \ldots (1)$$

i.e. a parabolic relationship between population size and equilibrium yield,

$$U_E = U_\infty - \left(\frac{q}{k} U_\infty\right) f \quad \ldots (2)$$

i.e. a linear relationship between fishing effort and catch per unit effort, and

$$Y_e = U_\infty f - \left(\frac{q}{k} U_\infty\right) f^2 \; \ldots (3)$$

i.e. a parabolic relationship between fishing effort and yield.

GULLAND (1961) and GARROT (1968) found a curvi-linear relationship, i.e. an exponential form between CPUE and fishing effort, instead of linear form as Eq.(2).

FOX (1970) uses the Gompertz growth model instead of logistic in describing the biomass regeneration in surplus yield models. The asymmetry of that model is more realistic growth in weight as expressed in

$$W = W_\infty e^{-k(e^{-ct})}$$

where *t* is the age of the fish, *k* is the growth coefficient, and *c* is a constant. This equation approaching to an asymptotic wight W_∞ when *t* approaching to infinity ∞.

In equilibrium conditions, the surplus production of FOX (1970) can be defined as

$$\frac{dB}{dt} = kB(\log_e B_\infty - \log_e B) - qfB = 0$$

or equilibrium yield is

$$Y_E = qfB = kB(\log_e B_\infty - \log_e B) \; \ldots \; (4)$$

Since CPUE *U* is proportional to biomass *B* by definition (i.e. U = qB), Eq. (4) can be expressed as

$$Y_E = k\frac{U}{q}\left(\log_e \frac{U_\infty}{q} - \log_e \frac{U}{q}\right)$$

since Y_E = fU, then

$$fU = \frac{k}{q} U \left(\log_e \frac{U_\infty/q}{U/q}\right)$$

deviding through U, then

$$f = \frac{k}{q}(\log_e U_\infty - \log_e U)$$

$$\log_e U = \log_e U_\infty - \frac{q}{k} f$$

or

$$U - U_\infty e^{-(q/k)f} \quad \ldots \quad (5)$$

Multiplying Eq. (5) by the annual fishing effort *f*, becomes

$$Y_E = f\, U_\infty\, e^{-(q/k)f} \quad \ldots \quad (6)$$

The three relationships of Fox model that can be compare to that of Graham-Schaefer model (i.e. equations (1), (2), and (3) are:

$$Y_E = kB(\log_e B_\infty - \log_e B)$$

i.e. relationship between biomass and equilibrium yield,

$$U = U_\infty e^{-(q/k)f}$$

i.e. relationship between fishing effort and CPUE, which is a declining curve, and

$$Y_E = f\, U_\infty e^{-(q/k)f}$$

i.e. relationship between fishing effort and equilibrium yield.

From the exponential expression of the surplus yield model, the following properties can be described:

(1). The optimum of the fishing effort $f_{(opt)}$ that produces the maximum equilibrium yield. $Y_{E\,(max)}$ is obtained by taking differential of Eq. (6) with respect to *f* and equating to zero:

$$\frac{dY_E}{df} = U_\infty e^{-(q/k)f} + f U_\infty e^{-(q/k)f}(-q/k) = 0$$

$$\frac{q}{k} f = 1$$

$$f_{(opt)} = \frac{k}{q} \quad \ldots \quad (7)$$

(2). The CPUE at MSY, $U_{(opt)}$ can be obtained from Eq. (5) by putting the value of $f_{(opt)}$:

$$U_{(opt)} = U_\infty e^{-(q/k)(k/q)} = U_\infty e^{-1}$$

$$U_{(opt)} = \frac{U_\infty}{e} \quad \ldots \quad (8)$$

(3). The maximum equilibrium yield $Y_{E(max)}$ is

$$Y_{E(max)} = U_{(opt)}\, f_{(opt)}$$

$$= (k/q)\frac{U_\infty}{e} \quad \ldots \quad (9)$$

Although it is usually not essential to know rate of fishing and stock size, these can be estimated if an estimate of catch-ability coefficient *q* is at hand, for example obtained from tagging experiment. The optimal rate fishing is

$$F_{(opt)} = q f_{(opt)} = q \cdot \frac{k}{q} = k$$

Stock size required for MSY :

$$B_{(opt)} = \frac{Y_{E(max)}}{F_{(opt)}} = \frac{k}{q} \cdot \frac{U_\infty}{e} : k$$

$$B_{(opt)} = \frac{q B_\infty}{q \cdot e} = \frac{B_\infty}{e} \quad \ldots \quad (10)$$

The maximum stock size, $B_\infty = \dfrac{U_\infty}{q}$

Parameters Estimation

As an example, let us use the data of catch and effort of the lemuru fishery (oil sardines, *Sardinella longiceps)* as illustrated on Table 1 of WIDODO (1986). Exponential production model of Fox defines that CPUE is an exponential function of fishing effort, which in general can be expressed by:

$$y = ae^{-bx}$$

where y and x are CPUE and fishing effort in the same year respectively, a is U_∞ and $b = q/k$. Fitting y on x would got the estimation of the relationship between CPUE and fishing effort, that is

$$U = 462.47\ e^{-0.01\ f}$$

which means U_∞ = 462.47 tons and q/k = 0.01.

The optimum fishing effort $f_{(opt)}$ which produces MSY can be defined from Eq.(7):

$$f_{(opt)} = k/q = 100.$$

From Eq. (8) we get

$$U_{(opt)} = \frac{U_\infty}{e} = \frac{462.47}{e} = 170.13$$

MSY calculated from Eq. (9) as

$MSY = U_{(opt)}.f_{(opt)}$

170.13 x 100 = 17 013 tons/year.

The curves of the relationship between CPUE and fishing effort as well as yield and fishing effort are illustrated in Fig. 1.

Note: The values of $f_{(opt)}$ and *MSY* in this calculation do not reflect the real situation of the lemuru fishery in the Bali Strait, it just only a numerical example of fitting the Fox model.

GENERALIZED PRODUCTION MODEL-PELLA AND TOMLINSON (1969)

Equilibrium yield Y_E as a function of biomass in Graham-Schaefer model can be expressed as

$$Y_E = kB - \frac{k}{B_\infty} B^2 \quad \ldots \quad (11)$$

which is a symmetric parabola. PELLA and TOMLINSON (1969) expressed in a more general form, in which the exponent 2 in Eq. (11) is replaced by a variable m (Ricker 1975), and becomes

$$Y_E = kB - \frac{k}{B_\infty^{m-1}} B^m \quad \ldots \quad (12)$$

In its original form PELLA & TOMLINSON model (1969) is expressed as:

$$C = -KP - \frac{K}{H} P^m$$

where $C = Y_E$, $P = B$, $H = \frac{k}{B_\infty^{m-1}}$, and

$$K = -k.$$

Pella and Tomlison model as defined in Eq. (12) gives the results that $Y_{E(max)}$ or MSY can be associated with any value of B, which being restricted with $B_\infty/2$ as that of Graham-Schaefer model.

When m = 2, we have got the Graham-Schaefer model, i.e. the plot of yield on biomass is a symmetrical parabola. If m < 2, such yield curve is asymmetrical parabola with maximum displaced toward the origin, if m > 2 the maximum of the asymmetrical curve is displaced away from the origin.

Numerical explanation :

If m = 2, then Pella and Tomlinson model, i.e. Eq. (12) becomes

$$Y_E = kB - \frac{k}{B_\infty} B^2$$

which is the Graham-Schaefer model.

If m < 2, say m = $1^1/_2$, Eq. (12) becomes

$$Y_E = kB - \frac{k}{B_\infty^{1½}} B^{1½}$$

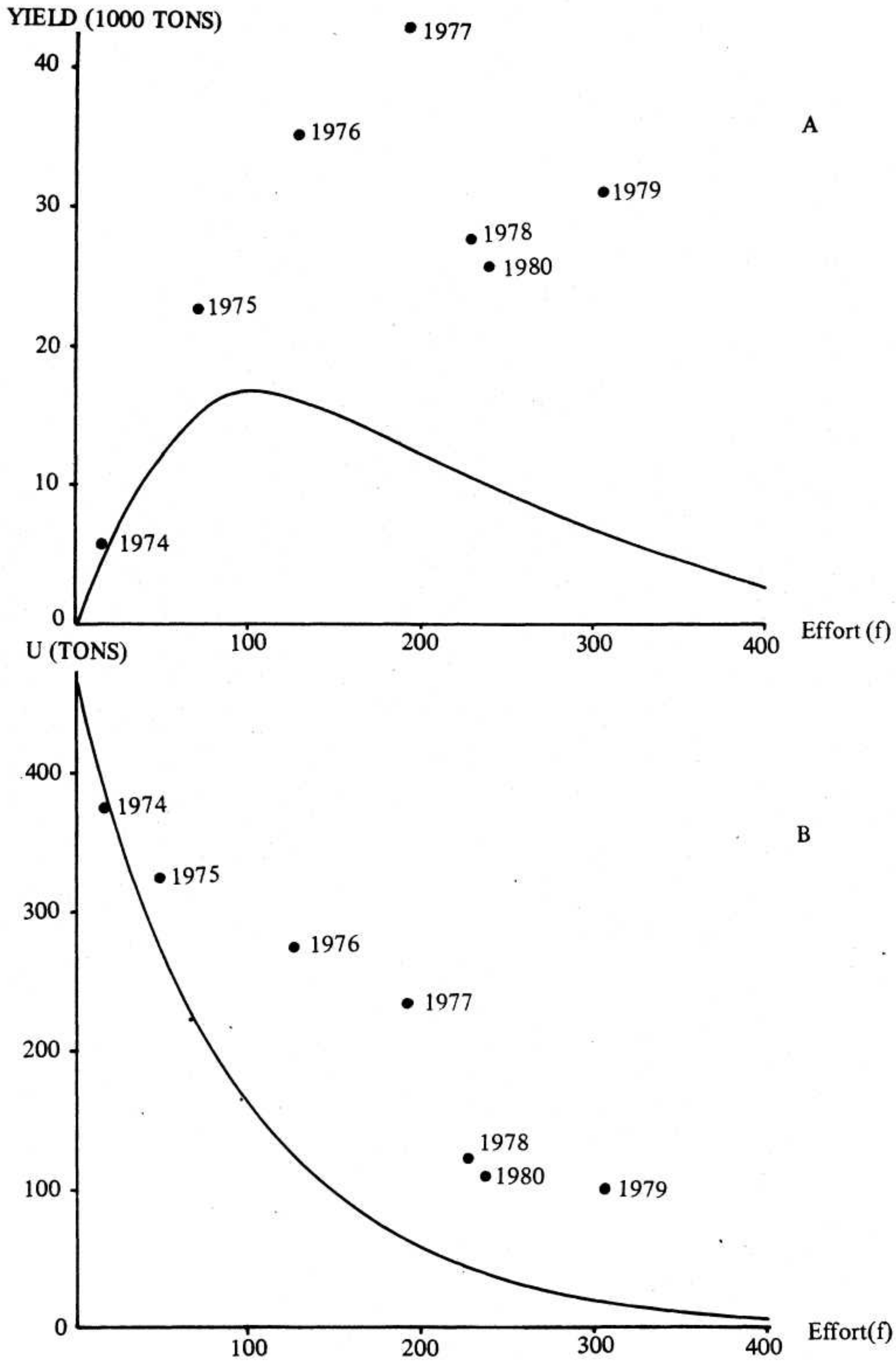

Figure 1 : Fox model (1970). A. Relationship between yield and effort. B. Relationship between CPUE and effort.

To get its maximum by differentiating with respect to B, then equating to zero

$$\frac{dY_E}{dB} = k - \frac{k}{B_\infty^{1½}} \cdot 1½ \cdot B^{(1½ - 1)} = 0$$

$$B_{(opt)} = \frac{4}{9} B_\infty$$

i.e. the maximum is displaced toward the origin.

If m> 2, let m = 4;

$$Y_E = kB - \frac{k}{B_\infty^3} B^4$$

differentiating with respect to B and equating to zero

$$\frac{dY_E}{dB} = k - 4\frac{k}{B_\infty^3} B^3 = 0$$

$$B^3 = \frac{B_\infty^3}{4}$$

$$B = 0.65\, B_\infty$$

i.e. the maximum is displaced away from the origin.

In other words, plotting either yield and biomass or yield and fishing effort will result in parabola, with its maximum depends upon the value of m. The left line is steeper than the right when m < 2, the left line is less steep if m > 2, and finally the parabola is symmetric when m = 2 .

It is necessary to use a computer program, since there are four parameters to be estimated (i.e. k, B_∞, q and m), and numerous iterations are necessary. PELLA & TOMLINSON (1969) set up program GEN-PROD which they modified it later to GENPROD-2 in ABRAMSON (1971). Although good fits to observed data can be obtained, unless constrains are put on the value of m, the other parameters are frequently unreasonable. Consequently, this model has not as yet proven useful in practice.

CONCLUSIONS

The Graham-Schaefer model characterized by the requirement that MSY is reached when B = 50% of B_∞, and that CPUE is a linear function of fishing effort. Usually, the decrease of CPUE with fishing effort is not linear, and a better fit is given by an exponential decrease of CPUE (GULLAND 1961; GARROT 1968; FOX 1970), i.e. as expressed in Eq. (5).

It is important to realise that the position of MSY in relation to B_∞ is fixed for a particular alternatives of the surplus production models. For Graham-Schaefer model, MSY is always at $B_\infty/2$, i.e. at the middle of the symmetric parabola. In the same manner, the Fox model has as rigid a form as the Graham-Schaefer model, i.e. MSY produced by the population size of $\frac{1}{e}$ or 37% of the maximum biomass B_∞.

Although the Pella and Tomlinson model, characterized by the assumption that MSY can be associated with any value of B lying in the interval from 0 to B_∞, however, once the additional parameter m is fitted, MSY and $f_{(opt)}$ will fixed in relation to B_∞. So that the only 'general' thing about Pella and Tomlinson model is that the biomass regeneration function may assume a variety of shapes, but not all possible shapes, taking into account, for instance, a minimum viable stock size and another internal constraints on parameter values (PITCHER & HART 1982; RIVARD & BLETSOE 1978).

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