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SURPLUS PRODUCTION MODELS AND ANALYSIS OF EXPLOITED POPULATION IN FISHERIES

by

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ABSTRAK

MODEL 'SURPLUS PRODUCTION' DAN ANALISIS TERHADAP POPULASI DI BIDANG PERIKANAN. *Didalam ilmu dinamika populasi perikanan, kelompok model ini dapat digolongkan ke dalam model yang paling sederhana, dalam arti mudah dimengerti (meski oleh kaum awam sekdipun), tidak memerlukan banyak macam data, serta didasari oleh pengertian matematika yang mudah. Konsep-konsep yang melandasi model ini dibahas secara rinci, termasuk pula beberapa kelemahan dan keunggulannya bila diterapkan untuk melakukan analisis terhadap dinamika dari suatu populasi di bidang perikanan, Selain itu dikemukakan pula beberapa contoh penggunaan model tersebut, antara lain untuk perikanan lemuru di Selat Bali.*

INTRODUCTION

Fisheries represent dynamic (time varying) system with interacting components, such as biology, technology, economy, politic, and social. Mathematical models in fisheries try to capture how a system works by expressing the interactions in terms of mathematical relationships.

The simplest model in fisheries population dynamics is the surplus yield, surplus production, Schaefer models, or logistic production models. Actually, this simple yield approach can be traced back to GRAHAM (1935), so that some authors prefer to name it as Graham-Schaefer model. These models consider a fish population as a single entity, subject to simple rules of simple population growth. Increasing or decreasing process in biomass subsumes a number of real population processes such as tissue growth and recruitment as input parameters and mortality as output parameter.

Consequently, analysis of fish population based upon these models can be made when only very little information, primarily on the catch, the population biomass, and the amount of fishing which usually expressed as fishing effort, are available. These models ignoring the events within a population and the growth and mortality of the individuals forming the population.

POPULATION GROWTH FORM

Before discussing the growth of a population, it is necessary to define the word population. In this paper, population defined as all collection group of fish of the same species inhabiting a particular space which enable them to interbreed independently. Each of population has its characteristic patterns of increase which called as population growth form. Two population growth forms, i.e. exponential growth and sigmoid growth forms, are important in the study of fish population dynamics.

1). Badan Penelitian Pengembangan Pertanian, Sub Balai Penelitian Perikanan Laut, Semarang.

1. Exponential Population Growth Form

Exponential growth form known also as J-shaped, geometric, or Malthusian population growth. This last name is derived fron that of THOMAS ROBERT MALTHUS (1766-1834) who pointed out that all species had, theoritically, an ability to increase that finally would exceed any conceivable increase in the means of subsistance of those species (PIELOU 1974).

Unchecked exponential growth of a population, i.e. the environment is unlimited in space, food, and other organisms not exerting a limiting effect, will lead to a *population explosion.*

Let us suppose that in general, at the end of any unit of time there are always X times as many individuals as there were at the be-ginning of the unit of time, and let

$$N_0, N_1, N_2, N_3, \ldots N_t$$

denote the size of the population at time

$$t = 0, 1, 2, 3, \ldots t$$

then,

$$N_1 = \lambda N_0$$

$$N_2 = \lambda N_1 = \lambda^2 N_0$$

$$N_3 = \lambda N_2 = \lambda^2 N_1 = \lambda^2 N_0$$

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$$N_t = \lambda^t N_0$$

Thus, the size of the population at the sequence of times is

0	1	2	3	...	t
N_0	λN_0	$\lambda^2 N_0$	$\lambda^3 N_0$		$\lambda^t N_0$

which demonstrate a geometric series. The constant λ is known as the finite rate of pupulation growth.

In the exponential population growth form, the population is growing exponen-tially or like a sum of money earning compound interest with interest compounded annualy. Then N_t, the size of the popula-tion after *t* years is

$$N_t = N_0 (1 + r)^t \ldots \quad (1)$$

where *r* is the interest rate expressed as a fraction.

Numerical example : let N_0 = 1 000 be the sum of muney (in rupiah) saved in bank at time *t* = 0, and *r* be the interest rate = 0.1.5 per year, then at time

$$t=1 \quad N_1 = 1\,000 + (1\,000 \times 0.15) = 1\,000(1 + 0.15) = N_0(1+0.15)$$

$$t=2 \quad N_2 = N_1 + (N_1 \times 0.15) = N_1(1 + 0.15) = N_0(1 + 0.15)^2$$

$$t=3 \quad N_3 = N_2 + (N_2 \times 0.15) = N_2(1 + 0.15) = N_0(1 + 0.15)^3$$

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$$t \quad N_t = N_{t-1} + (N_{t-1} \times 0.15) = N_{t-1}(1 + 0.15) = N_0(1 + 0.15)^t$$

Consider again the compound interest law of

$$N_t = N_0 (1+r)^t$$

If interest were compounded *n* times a year instead of annually, Eq. (1) would become

$$N_t = N_0\left(1 + \frac{r}{n}\right)^{nt}$$

and if *n* becomes very large, it may be shown that in the limit $N_t \longrightarrow N_0 e^{rt}$ where $e = 2.71828 \ldots$ is the base of the natural (or Napierian) logarithms.

The arithmetic plots of the exponential population growth demonstrate the J-shaped growth curve as shown in Figure 1. The con-stant *r* is known as the instantaneous rate of

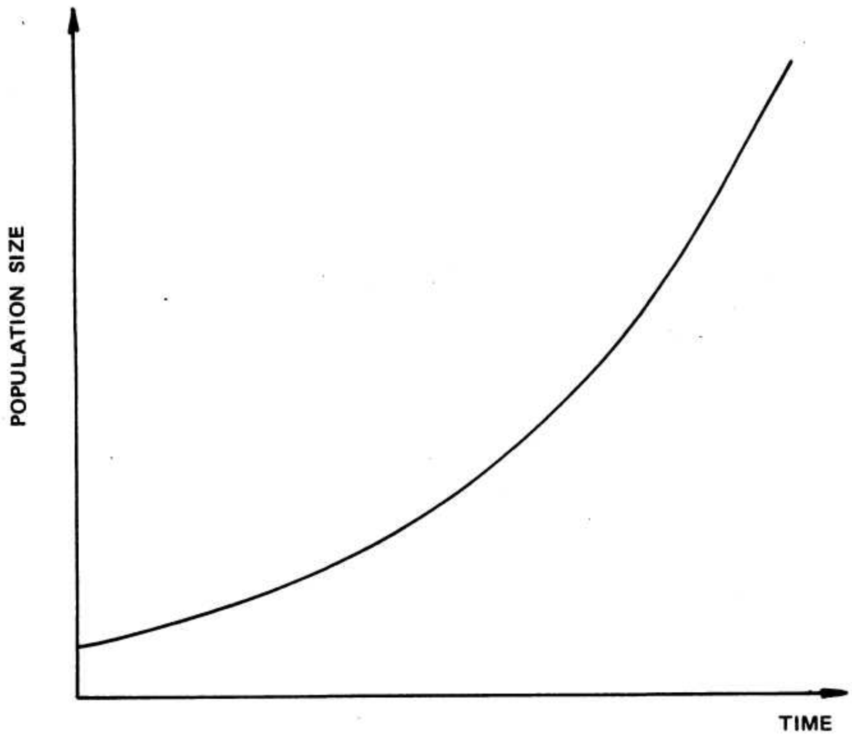

Figure 1. Exponential growth curve.

population growth. Thus the relation between λ and r is this : in so far as the equations

$$\frac{Nt}{N0} = \lambda^{t} \quad \text{and} \quad \frac{Nt}{N0} = e^{rt}$$

are equivalent, it follows that $\lambda = e^r$ or r = In λ, where ln λ denotes $\log_e \lambda$, the natural logarithm of λ. The rate of increase r is a composite parameter reflecting the difference between birth rate b, and death rate d, such that r = b - d (these are instantaneous rates).

2. Sigmoid Population Growth Form

The abundance of a fish species is not a fixed quantity and it varies from one place and one time to another, resulting spatial and temporal patterns. Fluctuations are subject to the changing balance between death and birth rates and by the availability of resources. The other simple population growth form that we usually can study is the sigmoid curve or S-shaped form which describes the way in which the size of a pupulation approaches an asymptotic and fluctuates about it as the relationship changes between births and deaths. Mathematically, the sigmoid growth form can be expressed in a differential equation as

$$\frac{dN}{dt} = rN\left(\frac{K - N}{K}\right).....(2)$$

where $\frac{dN}{dt}$ = the rate of population growth change (in number in time), K is the population abundance which can be supported by the environment (the carrying capacity), N is the population size, and r is the specific growth rate. The shape of the curve describing the growth of N, assuming the sigmoid curve is valid, is shown in the Figure 2.

Starting from a low N, abundance increase slowly at first, then faster until the rate of increase drops away as N approaches K. The carrying capacity is the population abundance towards which the population converges as equilibrium is disturbed.

The term of $(\frac{K - N}{K})$ in Eq. (2) describes detrimental factors created by the growing population itself, i.e. whenever N increases, $\frac{dN}{dt}$ decreases. In general, sigmoid curve characterized by the greater and greater detrimental factors as the abundance of the population increases. The growth to be *logistic* whenever detrimental factors are linearly proportional to the population size.

SURPLUS PRODUCTION MODELS

The basis of the production models is biomass regeneration, which considered as a single entity process, ignoring the events of recruitment, growth, and mortality of the individuals composing the population. These models assume that biomass produced over that needed for exact replacement is regarded as a *surplus* which can therefore be harvested. This first assumption can be depicted in Figure 3, where B(t) and B(t+1) are biomass at time t and t+1, B_∞ is the biomass maximum, and Y_E is the equilibrium yield.

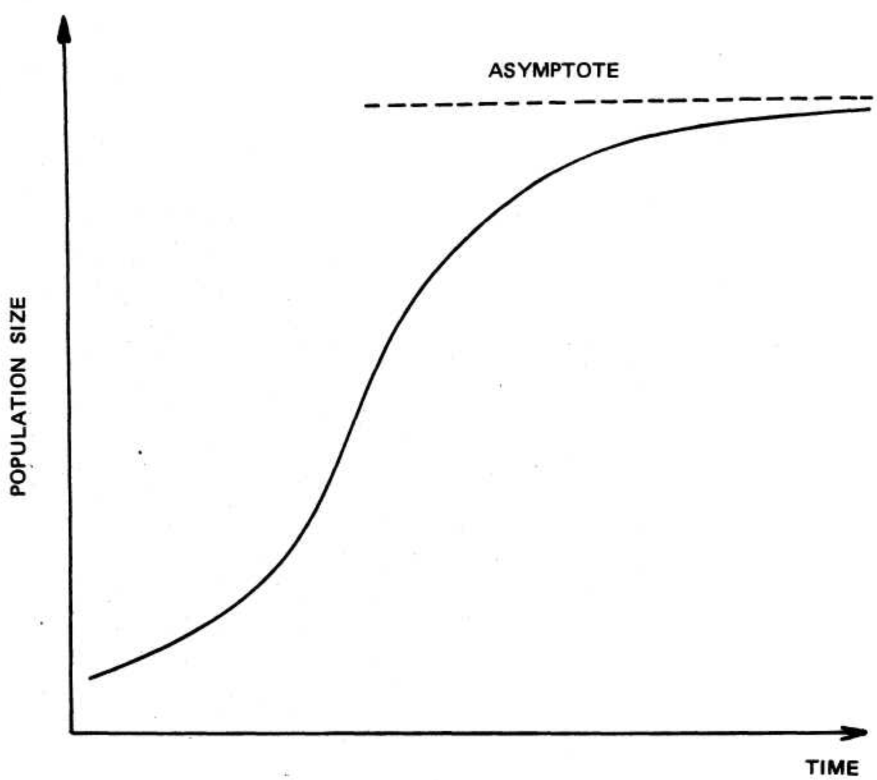

Figure 2. Sigmoid growth curve.

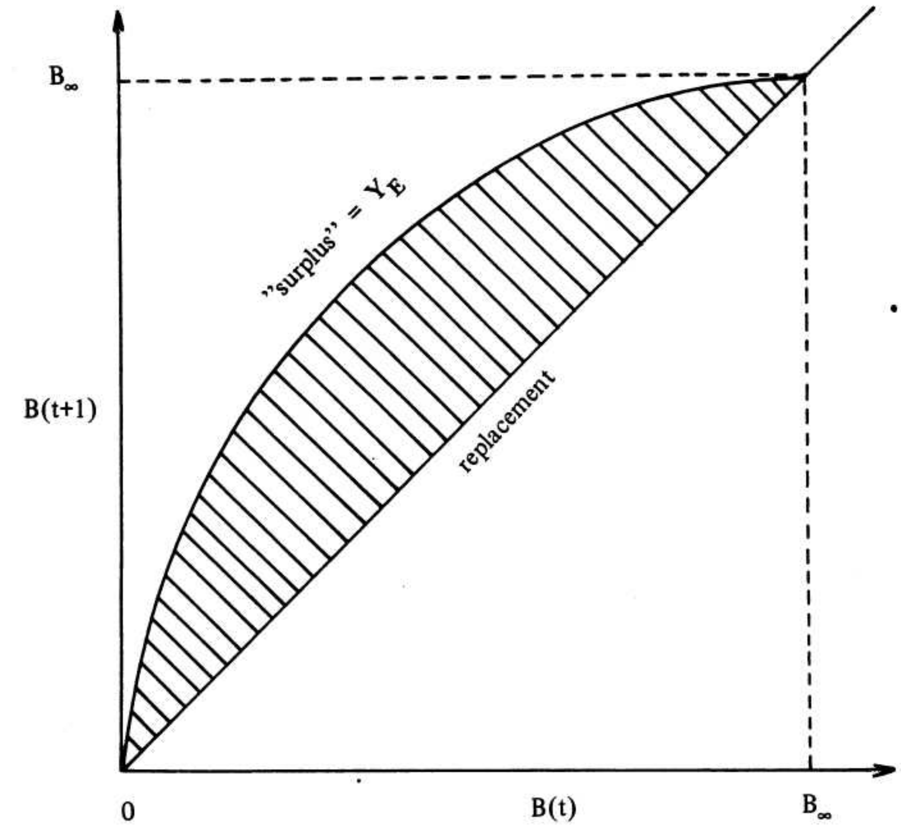

Figure 3. Biomass of time = t plotted against biomass of time = t+1 to demonstrate the basic assumption of surplus production models.

The second assumption is that when the quantity of biomass taken in the fishery is exactly equal to the surplus produced, the fishery is assumed to be in steady state (equilibrium), providing an equilibrium yield, Y_E.

1. Parabolic Surplus Production Curve — GRAHAM'S Method

GRAHAM (1935) postulated that under steady state conditions the logistic growth equation could apply to the biomass regeneration function, i.e. the rate of surplus production of stock (= recruitment + growth less natural mortality) is directly proportional to its biomass and also to the difference between the actual biomass and the maximum biomass the area will support

$$\frac{dB}{dt} = kB\left(\frac{B_\infty - B}{B_\infty}\right)........(3)$$

where $\frac{dB}{dt}$ the rate of surplus production of the stock

B stock size (biomass)

B_∞ maximum biomass that could be supported by the environment.

k the instantaneous growth rate at small biomass.

t time, conventionally in year.

Note: Integrating Eq. (3), the growth curve is the sigmoid logistic curve Verhulst (RICKER 1975)

$$B = \frac{B_\infty}{1 + e^{-k(t-t_0)}}$$

where t0 is the inflection point of the curve, i.e. $t - t_0 = 0$ when $B = \frac{B_\infty}{2}$. Therfore, surplus production models also well known as logistic production models.

Based upon the second assumption, i.e. when fishing remove the surplus production of the stock at the same rate as it is produced, it becomes the annual yield from a stock held in equilibrium, mathematically can be expressed as

$$\frac{dB}{dt} = k B_E\left(\frac{B_\infty - B_E}{B_\infty}\right) - F_E B_E = 0$$

$$Y_E = F_E B_E = k B_E \left(\frac{B_\infty - B_E}{B_\infty}\right)$$

$$Y_E = k B_E - \frac{k B_E^2}{B_\infty} \quad \ldots \quad (4)$$

where B_E biomass of stock in steady state conditions.

F_E rate of fishing which maintain the stock in equilibrium.

Y_E yield when the stock is in equilibrium.

Eq. (4) demonstrates that the relation between equilibrium yield and equilibrium biomass is a parabola, i.e. Y_E is a parabolic function of B_E as shown in Figure 4. The parabolic production curve has the intercepts with horizontal axis at BE = 0 and

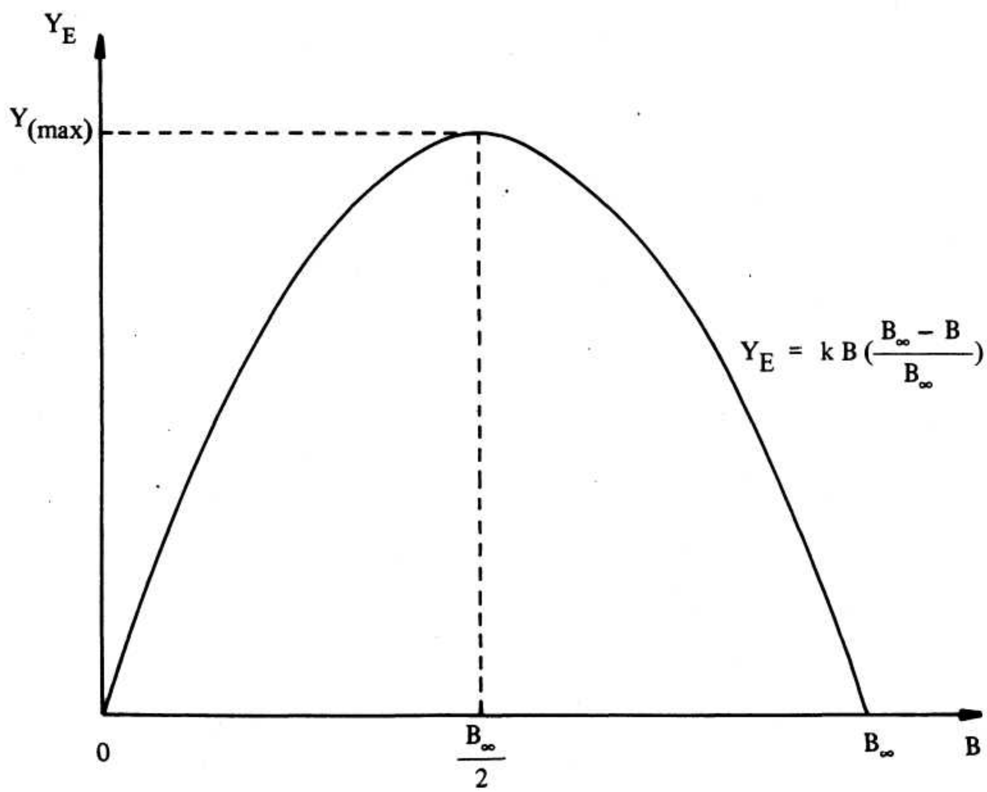

Figure 4. Equirilibrium yield (Y_E) against biomass

$B_E = B_\infty$. To obtain maximal/minimal of this curve, basic calculus can be employed, i.e. by differentiating Eq. (4) and equating to zero resulting

$$\frac{dY_E}{dB_E} = k - 2\frac{k}{B_\infty} B_E = 0$$

$$B_{(opt)} = \frac{B_\infty}{2}$$

substituting the value of B_∞ into Eq. (4) giving

$$Y_{(max)} = k\frac{B_\infty}{2} - \frac{k}{B_\infty}\left(\frac{B_\infty}{2}\right)^2$$

$$Y_{(max)} = k\frac{B_\infty}{4}$$

As a result, the maximum equilibrium yield or MSY is derived when the biomass is exactly half of the maximum equilibrium biomass, and it is equal to one-quarter of the maximum biomass multiplied by the instantaneous rate of increase at very low level of biomass ($MSY = \frac{kB_\infty}{4}$).

2. Application/Fitting to Data

It is desirable to use method of fitting this model to the observed data, which makes the most effective use of the observations, resulting conclusions of useful advice and on the contrary avoiding of misleading guidance.

This numerical example is freely modified from RICKER (1975) which illustrates the fitting of the Graham's method, given the value of B_∞ and equilibrium fishing effort (p.312). An equilibrium conditions of a fishery was characterized by Y = 40 000 tons/year, of which 30 000 tons were fish of vulnerable size at the beginning of the year. From mark-recapture experiment, the rate of exploitation was found to be 30 percent. The catchable stock present at the beginning of the year was defined to be 30 000/0.30 = 100 000 tons. As the fishery was in steady state conditions, this represents also the equilibrium vulnerable stock, B_E. The rate of fishing, F_E was equal to 40 000/100 000 = 0.40, and this must also be the natural logistic growth rate, i.e. rate of recruitment plus rate of growth less rate of natural mortality.

Catch per unit effort was currently known as 10 tons/boat-day. But, a few years earlier, soon after a long of no-fishing period, catch was 22 tons/boat-day. Considering that Y/f (catch per unit effort) is proportional to stock (CPUE as an index of relative abundance), therefore

$$\frac{22}{10} = \frac{B_\infty}{B_E}$$

$$B_\infty = 22/10 \times 100\,000 = 220\,000$$

tons. Substituting this value of B_∞ into Eq. (4) we get

$$40\,000 = k \times 10\,000\left(\frac{220\,000 - 100\,000}{220\,000}\right)$$

from which k = 0.77. Yield as a function of biomass in the steady state condition of this fishery can be expressed as

$$Y_E = 0.77\, B_E - \frac{0.77}{20\,000} B_E^2$$

3. Relation of CPUE to Fishing Effort - SCHAEFER's Method

So far the model of Graham may not directly helful. By a mathematical manipulation, SCHAEFER (1954) was successfully modified the model in terms of directly useful to fishery managers and at the same time it could be fitted using the real data of catch, abundance, and number of fishing easily and routinely collected by the same managers (PITCHER & HART 1982).

We can define the fish catch per unit effort at equilibrium, U_E as

$$U_E = Y_E/f \quad \ldots \quad (5)$$

where / is fishing effort. The unit chosen for expressing effort do not matter as long as they remain consistent. In so far as we can define yield as biomass times the rate of fishing times catchability

$$Y_E = f\, q\, B_E \quad . . . \quad (6)$$

from Eq. (5) and Eq. (6) we can express that

$$U_E = f\, q\, B_E / f = q\, B_E$$

and can therefore express

$$B_E = U_E / q$$

i.e., stock size expressed as CPUE over catchability. Sinse the relationship is true only at equilibrium conditions, it cannot really be used to predict stock size when data of CPUE and catchability coefficient are available. This equation just demonstrates the fact that CPUE will change at different biomass level. The trick recognized by SCHAEFER is to substitute this new value for B and divided by U_E in Eq. (4) to get, for the SCHAEFER's model

$$f\, q \frac{U_E}{q} = \frac{U_E}{q} \left(\frac{B_\infty - (U_E/q)}{B} \right)$$

divided by U_E

$$f = \frac{k}{q} \left(1 - \frac{U_E}{q B_\infty} \right)$$

as $qB_\infty = U_\infty$, then

$$f = \frac{k}{q} \left(1 - \frac{U_E}{U_\infty} \right)$$

which can be easily expanded and rearanged to solve for U_E, giving

$$U_E = U_\infty - \left(\frac{q}{k} U_\infty \right) f \quad . . . \quad (7)$$

Eq. (7) shows the general form of a linear regression of CPUE as a function of fishing effort *f*, in the form of $y = a + bx$, where $a = U$ and $b = - \frac{q}{k} U_\infty$. Graphically, Eq. (7) can be illustrated in Figure 5 which describes a useful expression showing the relationship between CPUE and fishing effort.

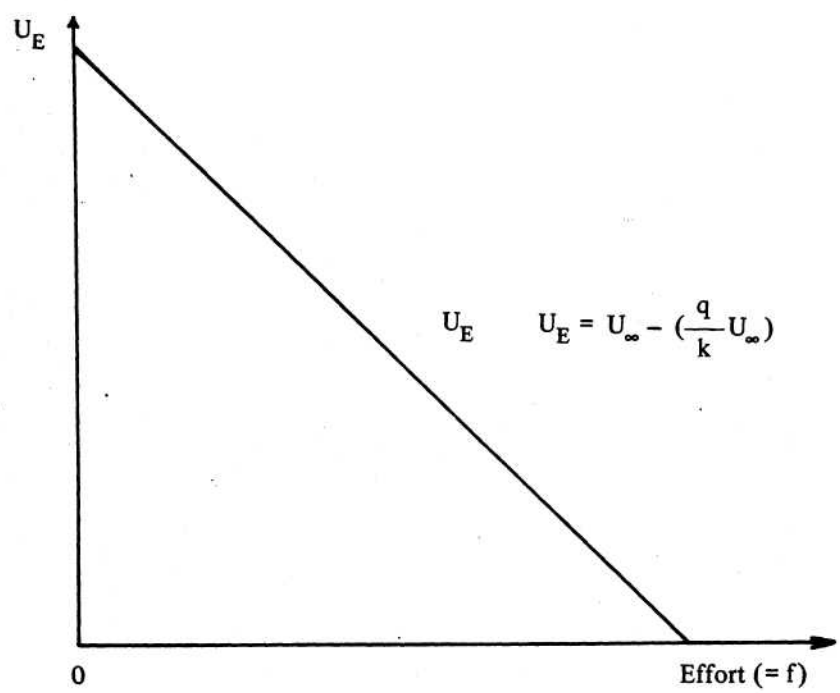

Figure 5. Catch per unit effort against effort.

4. Relation of Equilibrium Yield and Fishing Effort - SCHAEFER's Method

We use the results of the regression fitting of CPUE and fishing effort *f* in the final stage which purport to give the fishery manager exactly what be wanted, i.e. by relating fishing effort directly to yield.
Since $Y_E = U_E f$ by definition, we can get the SCHAEFER's model from Eq. (7) as

$$Y_E = U_\infty f - (\frac{q}{k} U_\infty) f^2 \quad \ldots \quad (8)$$

i.e. yield in equilibrium is a parabolic function of effort, which in general can be expressed by $y = af - bf^2$. By employing simple basic calculus the maxima and minima of the curve can be determined :

$$\frac{dy}{df} = a - 2bf = 0$$

$$f_{(opt)} = \frac{a}{2b} \quad \ldots \quad (9)$$

$$Y_{(max)} = a\frac{a}{2b} - b\frac{a^2}{4b^2}$$

$$Y_{(max)} = \frac{a^2}{4b} \quad \ldots \quad (10)$$

where $a = qB_\infty = U_\infty$ and $b = \frac{q}{k} U_\infty = \frac{q^2}{k} B_\infty$.

The maximum equilibrium yield or MSY and the optimum rate of fishing which produce the MSY simply can be obtained from the parabolic relationship between equilibrium yield and equilibrium fishing effort, as depicted in Figure 6.

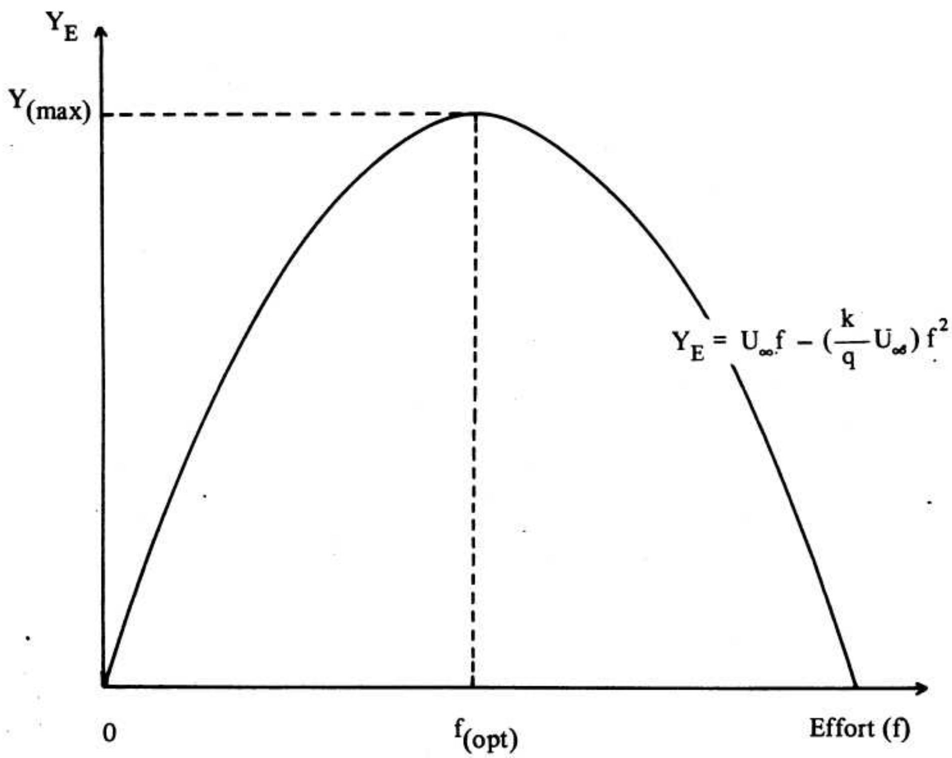

Figure 6. Equilibrium yield against fishing effort.

5. Numerical Example of Schaefer's Method

One approach to fitting data to this method is by manipulating the basic data, i.e. usually the annual catch and effort statistics, in such away that the procedure becomes a matter of fitting a simple curve to pair of values derived from these data, e.g. the CPUE in a given year and the effort of that year, and the yield versus the effort of the same year.

Let us take a look at the data of lemuru (oil sardines), *Sardinella longiceps,* of the Bali Strait as demonstrated by SUJASTANI & NURHAKIM (1982).

Tabel 1. Yield in tons, fishing effort in unit of purse-seiner, and yield per unit effort of lemuru (oil sardines), *Sardinella longiceps,* in the Bali Strait.

Year	Yield (tons)	Effort (unit of purse-seiner)	CPUE (Y/f) (ton/purse-seiner)
1974	6 380	17	375.3
1975	22 900	70	327.1
1976	35 204	126	279.4
1977	45 506	193	235.8
1978	27 915	228	122.4
1979	31155	304	102.5
1980	25 701	237	108.4

The yield (column 2) and the fishing effort (column 3) for each year are known and modified from fisheries statistics published by East Java and Bali Fisheries Agencies. There are divided to obtain the yield per unit effort (column 4), considered as an index of population abundance present each year.

Eq. (7) indicates the way to fit a curve to the lemuru data, i.e. by regressing U_E(= Y_E/f) on /. The estimate of the functional regression of U_E on / is as follows:

$$U_E = 400.1 - 1.06 f$$

By substituting a = 400.1, and b = 1.06 into Eq. (9) optimum fishing effort can be estimated as

$$f_{(opt)} = \frac{400.1}{2 \times 1.06} = 188.7$$

$$f_{(opt)} = 189$$

The estimation of the maximum yield can be carried out by putting the values of *a* and *b* to Eq. (10)

$$Y_{(max)} = \frac{(400.1)^2}{4 \times 1.06} = 37\,754.7$$

In conclusion, from the data illustrated in Table 1, MSY is estimated as 37 755 tons/year with estimation of optimal fishing effort of 189 units purse-seiner. The curves of the two relationships, i.e. CPUE vs. effort and Yield vs. effort can be illustrated in Figure 7.

CONCLUDING REMARKS

In general, the surplus production models require only simple data containing of one independent variable *t* (time), and four dependent variables, i.e. four function of time. Those are the population biomass B(t) with typical unit in ton, the rate of fishing effort *f(t)* in boat-day /year, the rate of catch *Y(t)* in ton/year, and the catch per unit effort *U(t)* in ton/boat-day. Besides, there are three parameters: the natural growth rate *k,* the carrying capacity B_∞, and the catchability coefficient *q.*

The axioms of the models consist of three equations:

$$\frac{db}{dt} = kb\left(1 - \frac{B}{B_\infty}\right) - 7 \quad \ldots \quad (11)$$

i.e. the total growth rate of population biomass equals its natural logistic growth rate minus the catch rate;

$$Y = q\,f\,B \quad \ldots \quad (12)$$

i.e. the catch rate is directly proportional to the effort rate and the available biomass with the constant of proportionality is *q* (the catchability coefficient);

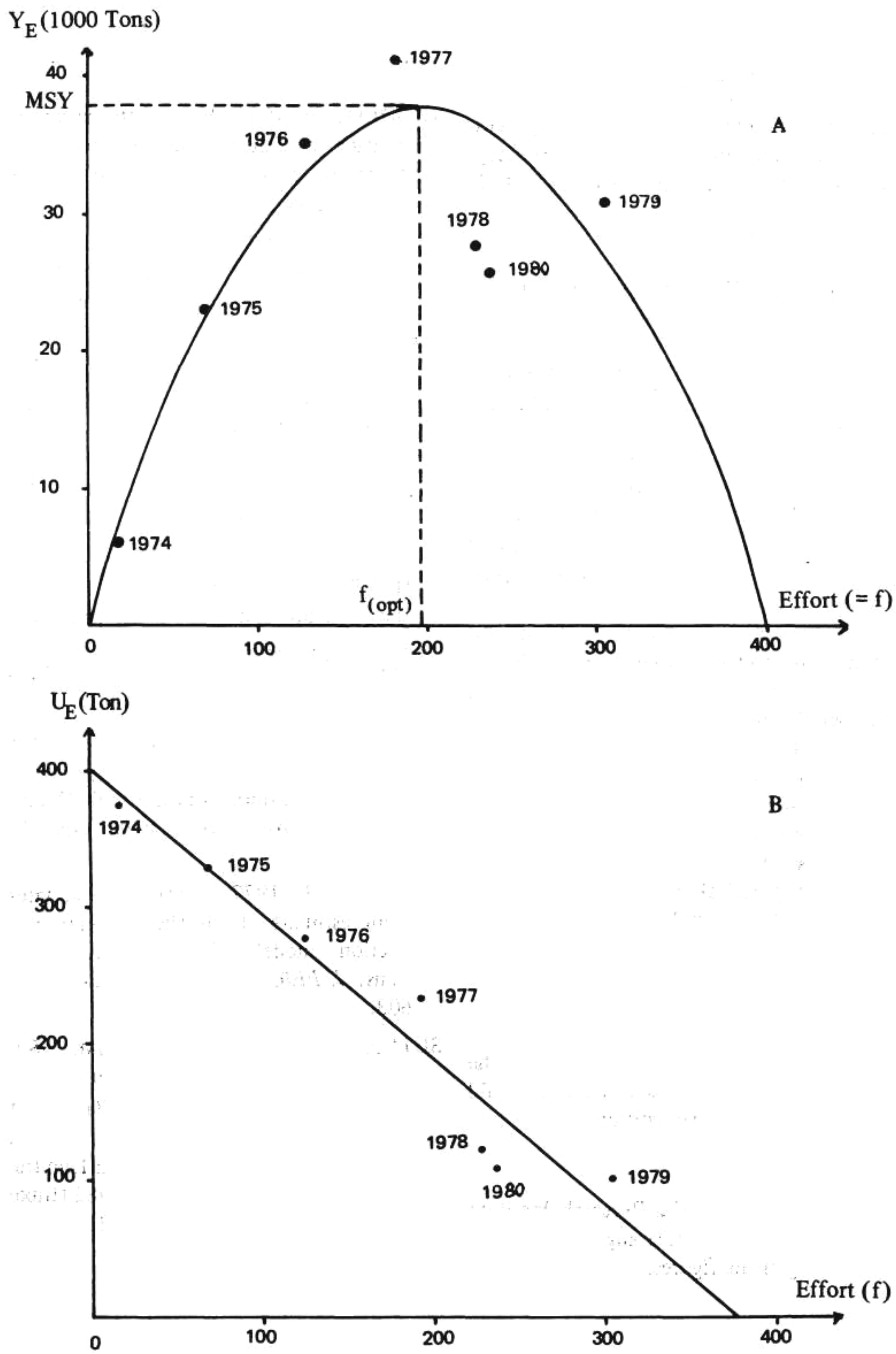

Figure 7. Graham-Schaefer model of the lemuru in Bali Strait.
A. Y_E vs. Effort B. U_E vs. Effort

$$U = q B \quad \ldots \quad (13)$$

i.e. the catch per unit effort itself proportional to the biomass alone. Actually, Eq. (12) and (13) imply that if $f \neq 0$, then

$$U = Y/f \quad \ldots \quad (14)$$

SCHNUTE (1977) points out that Eq. (14) should be the axiom, instead of Eq. (13). But, Eq. (14) is meaningful only when $f \neq 0$, while Eq. (13) is meaningful even when $f = 0$. From the Eq. (13) can be concluded that the CPUE, in general, is not zero even if the effort itself is zero. Accordingly, Eq. (13) suggests the correct interpretation for U, that is to say U is *potential* CPUE, namely, this potential is actualized only when fishing takes place, that is, when $f \neq 0$.

The main practical benefit of the surplus production models is that they require no demanding data, but catch and effort data over serial years. MSY is temptingly easy to calculate, in fact require no biologists to be employed in the fishery, and managers do not even have to get their feet and hands we in doing investigation on the actual fish (PITCHER & HART 1982). In contrast with their practical advantages as well as their attractive simplicity, these models ignore the real important biological processes that in fact generate the biomass of the population.

For this reason, the use of these unmodified simple production models in the management of. exploited fish stock should be employed with great caution.

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