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Performance Evaluation Of Two Ways Urban Traffic Control System Based On Macroscopic Hybrid Petri Net Model

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***Abstract*— Urban Traffic Control System (UTCS) is a system consists of interconnected roads, traffic regulations, vehicles and other entities that belongs in the road networking, systems management and traffic control system. This system has been created in order to achieve a safe journey of vehicles in a reasonable time. Nowadays, development of a reliable Urban Traffic Control System has become an urgent research topic. This paper describes model and control development of UTCS using hybrid petri net. Model of Two-ways traffic system has been developed and optimal control strategy has been implemented. As case, parameters of model has been obtained by measurement from real two ways intersection in South Jakarta. The effectiveness of UTCS using optimal control strategy has been verified by comparing to fixed control strategy using simulation.**

I. Introduction

URBAN TRAFFIC CONTROL SYSTEM (UTCS) consists of interconnected roads, vehicles, traffic regulations, traffic management and control system. The UTCS aims to construct a safe trip in a reasonable time [1]. Generally, UTCS consist of sensors, which can be used to estimate the traffic conditions, develop a control strategy and implemented in a traffic lights. In a large and complex urban area road network, a coordinated control strategy between intersections is required.

The UTCS development require a model of traffic system. Urban traffic system models are divided into two models, namely microscopic and macroscopic models [2]. Microscopic models study the behaviour of each vehicle movement and its interaction with other vehicles on the road links. On the other side, macroscopic models describe the dynamics of traffic flow as a collection of vehicles in a specific road link. The behaviours of macroscopic models is shown by the variables, i.e. the traffic density (number of vehicles/road distance unit), the average speed of vehicles in a specific road links (road distance unit/time unit), and the flow rate (number of vehicles passing/time unit). The relationship between macroscopic parameters on a specific road links derived from the fundamental traffic diagram. Afterwards, the modelling and simulation of a macroscopic urban traffic system is constructed using a *Petri net* approach. The Petri-net modelling get more attention of many researchers for discrete event dynamic modelling, nowadays [18].

This research was focused on the development of hybrid petri net models for UTCS. The hybrid Petri net models had been developed by Júlvez & Boel [3][14]. The vehicle flows are modelled by the continuous Petri net, while the traffic light system are described by discrete Petri net models, therefore the overall model obtained is a combination of both that is called hybrid Petri net. However, the developed Júlvez & Boel models are appropriate only for two one-way intersection traffic system and the traffic lights are not modeled explicitly. The actual condition of urban traffic system requires a more complex UTCS modelling.

The hybrid Petri net models of traffic system are improved in this paper by minimizing the objective function using an optimal control strategy. The control strategy is based on the modification of the traffic flow dynamics in a single intersection, which is controlled by adjusting the time duration of the traffic lights at each intersetion road arms. Through the adjustment, the objective function is accomplished, as

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indicated in the reduced total vehicle number at the intersection road arms for a certain period of time. furthermore, the adjustment of traffic lights observe the proportion of the stochastic fluctuations of the vehicles inflow into the intersection, so that the queueing of vehicles at each road intersection arms are spread optimal and the total waiting time of vehicles is minimized. The effectiveness of UTCS using hybrid Petri net model is verified by simulation. The performance is evaluated by comparing to fix control strategy. Performance evaluation is implemented in real case.

II. Basic Theory Of Hybrid Petri Net

Petri net structure is classified as a discrete Petri net and continuous Petri net. A hybrid Petri net structure contains both discrete and continuous one. A simple structure of a Petri net is shown in Fig. 1. A real structure of a urban traffic system modelled by Petri net is more complex.

(a) (b)

Fig. 1. Visual Representation of (a) Discrete PN and (b) Continuous PN

Petri net structure can be written in a set [4]:

$$\boldsymbol{PN} = (\boldsymbol{P}, \boldsymbol{T}, \boldsymbol{Pre}, \boldsymbol{Post}, \boldsymbol{m_0}) \quad (1)$$

Where in Fig. 1:

$\boldsymbol{P} = \{p_1, p_2, p_3, \dots, p_n\}$ denote sets of *Place* with $n > 0$; $\boldsymbol{T} = \{t_1, t_2, t_3, \dots, t_n\}$ denote sets of *Transition* with $m > 0$ in Fig. 1; $\boldsymbol{Pre}$ is the arc weight matrices with the size of matrices is ($\boldsymbol{P\ x\ T}$) and visualized by an arc from P to T; $\boldsymbol{Post}$ is the arc weight matrices, with the size of matrices is ($\boldsymbol{P\ x\ T}$) and visualized by an arc from T to P; $\boldsymbol{m_0}$ is the initial marking. Marking is visualized by a token (a black circle in a *Place* as in Fig. 1).

Place in a discrete Petri net (in a continuous Petri net, , respectively) is represented by a circle (a double circles) in Fig.1. *Transition* in a discrete Petri net (in a continuous Petri net, respectively) is represented by a bar (a double bar) in Fig.1.

The state dynamics of traffic system are modelled by the token placement and movement in the places. The token movement in Places follows the enabling- and the firing rules. Transition enabling rules are the ability/permission of token to move from one place to the next places. Transition firing rules mean the process of the movements of tokens from input place to output place. Firing occures only if the enabling rule is satisfied.

The value of transition enabling rules are called enabling degree. The enabling degree of a transition follow [5]:

$$enab(t,m) = \begin{cases} \min\limits_{p\in{}^\circ t} \left\lfloor \dfrac{m[p]}{Pre(p,t)} \right\rfloor & if\ t\epsilon T^d \\ \min\limits_{p\in{}^\circ t} \dfrac{m[p]}{Pre(p,t)} & if\ t\epsilon T^c \end{cases}$$

A transition *t* is said as *enabled* at the place p, if all places p, before the transition *t*, have markings or $enab(t,m) > 0$. An *enabled* transition $t\epsilon T$can fire in any amount α such that $0 \le \alpha \le enab(t,m)$. For a discrete Petri net ($t\epsilon T^d$), the place p contains at least number of markings equal to the weight of arc leading to the transition *t* ($\alpha\epsilon\mathbb{N}$), moreover, for a continuous Petri net ($t\epsilon T^c$), the number of markings must more than zero ($\alpha\epsilon\mathbb{R}$).

Firing of an *enabled* transition *t* is the moving process of the marking from the input places to the output places, in amount as much as the weight of the arc from the transition *t* to the output places. Such a firing lead to a new marking $m' = m + C(p,t).\alpha$, where $C = Post - Pre$ is the incidence matrix.

For time dependent petri net, the evolution of marking can be written as state equation as follows :

$$m(\tau) = m_0 + C.\sigma(\tau). \quad (2)$$

The term $\sigma(\tau)$ is the firing count vector at time τ. The firing count vector $\sigma(\tau)$ of a transition $t\epsilon T^c$ is differentiable with respect to time, and its derivative $f(\tau) = \dot{\sigma}(\tau)$ represents the *continuous flow* of transition *t*.

Under *infinite server* the flow of a transition $t\epsilon T^c$ is:

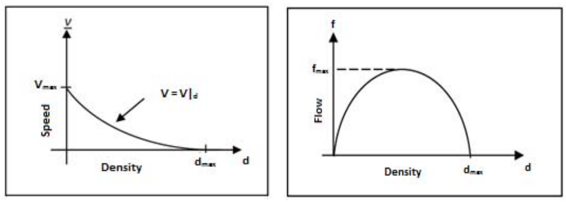
$$f[t](\tau) = \lambda[t].enab(t, m(\tau)) \quad (3)$$

while for the traffic light using *deterministic delay*, a transition $t\epsilon T^d$ with deterministic delay is scheduled to fire $1/\lambda[t]$ time units after it became enabled, where $\lambda[t] > 0$ is a constant parameter representing the internal speed of the transition.

III. Modeling Of traffic systems

A. Macroscopic Model of Traffic System

In this paper, macroscopic modelling approach is used to model the dynamics of urban traffic system. The dynamics models describe the traffic as a collection of vehicles in a specific road section. The dynamic of a spesific road section can be described by traffic density (number of vehicles/distance unit), the average speed of the vehicles, and the flow rate (the number of vehicles passing/time unit). These parameter dependecies of road section is generally represented by *fundamental traffic diagram* as in Fig. 2.



(a) (b)

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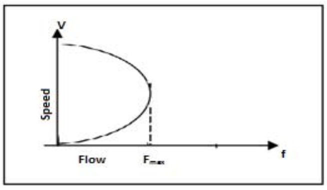

(c)

Fig. 2. (a) Speed-Density, (b) Flow-Density, (c) Speed-Flow Curve

B. Hybrid Petri Net Of Macroscopic Traffic System Model

Based on Júlvez & Boel [3], fundamental traffic diagram of a road section can be represented by continuous petri net as shown in Fig. 3.

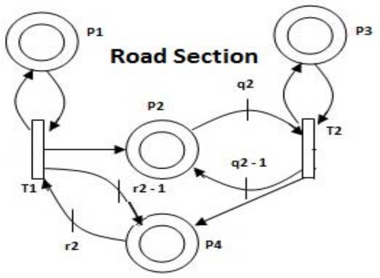

Fig. 3. Continuous PN for road section

Traffic lights are represented by discrete PN.

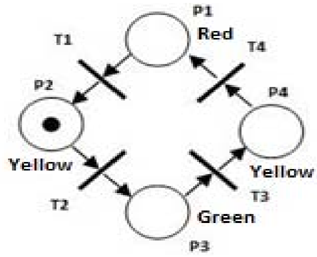

Fig. 4. Discrete PN for traffic lights

The urban intersection traffic system modelled in this paper is represented by hybrid Petri net (see Fig. 6).

Fig. 5. Pejaten Village's intersection maps

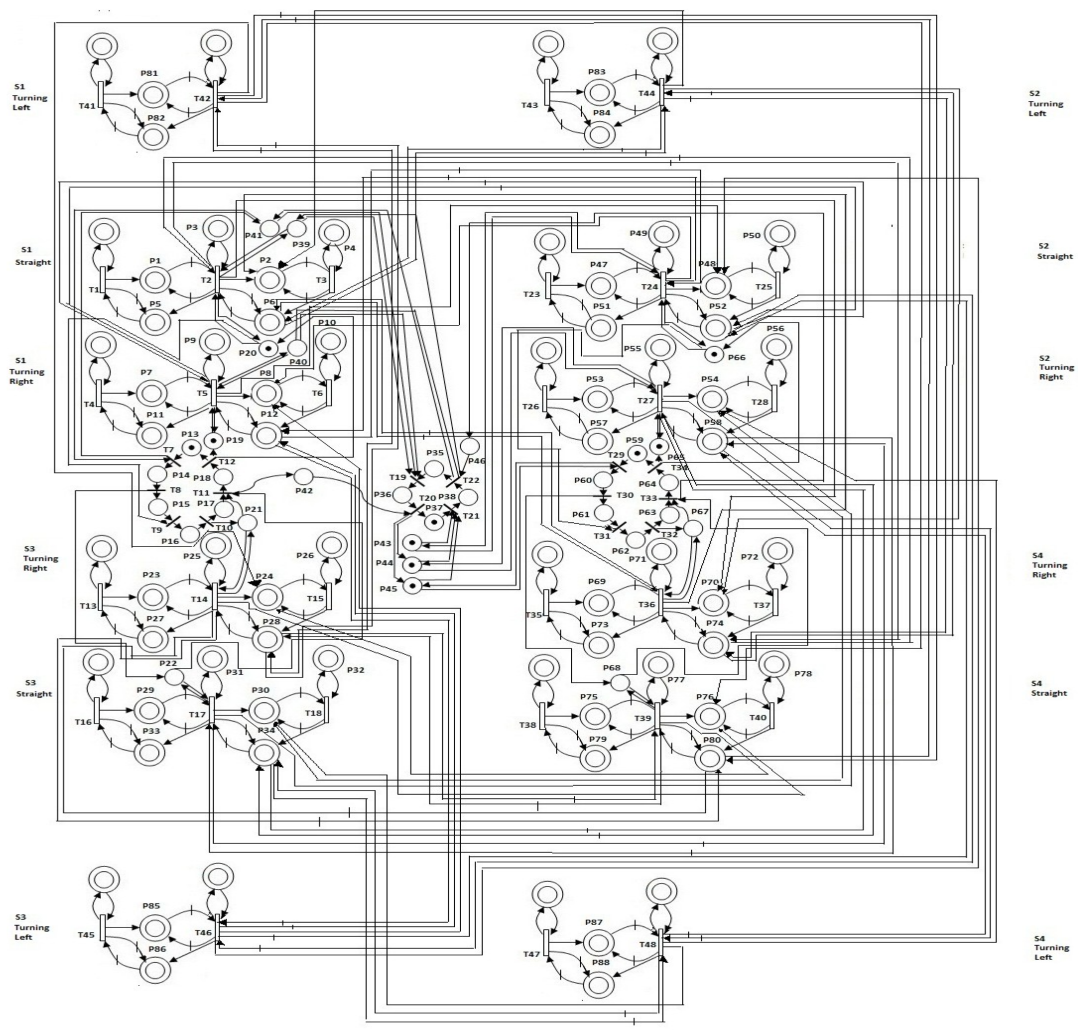

Fig. 6. Two ways intersection in *petri net*

IV. Simulation Of Two Ways Traffic System

A. Two Ways Intersection Model

The intersection model (see Fig. 6) represent a five phases of traffic flow sequences. These traffic flows are controlled by eight traffic lights. Each road section arm has two traffic lights to control the straight- and the turn right traffic flow movements.

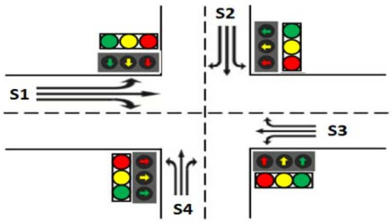

Fig. 7. Two ways intersection model

The traffic flow sequence arrangements of this intersection are :

1) the green lights are on for straight movement and turn right movement of S1,the rest are red.
2) the green lights are on for straight movement of S1 and S3,turning right movement of S1 and the rest are red.
3) the green lights are on for straight movement of S1 and S3,turning right movement of S3 and the rest are red.
4) the green lights are on for straight and turning right movement of S2 and the rest are red.
5) the green lights are on for straight and turning right movement of S4 and the rest are red.

The signalling time of traffic flow sequences for the intersection is ilustrated in Fig. 8.

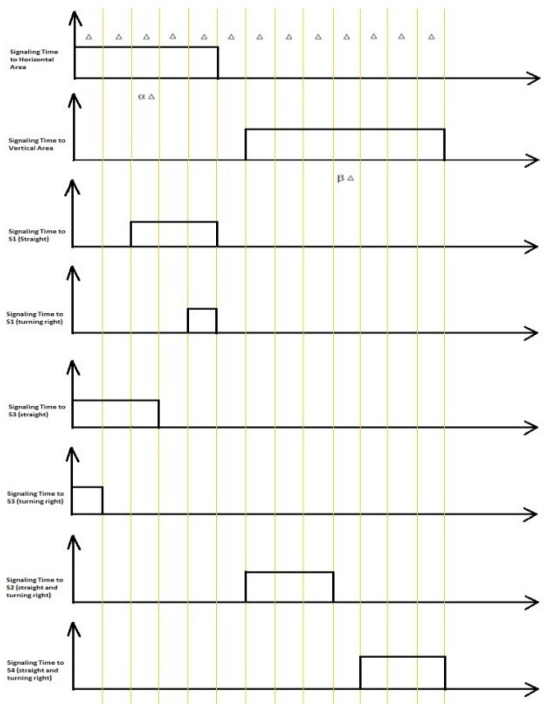

Fig. 8. Signaling time for traffic lights

The parameter α is the time period ratio for horizontal traffic flow (i.e. : S1 - S3 flow movement) and β is the time period ratio for vertical traffic flow (i.e. : S2 - S4 flow movement). The total time period for one cycle of traffic light in this intersection is (α+β+2)Δ. In the fix time control, the parameter of α and β is taken from the historical data of inflow into the road section.

V. Control Strategy

Optimal control strategy is proposed in this paper, where traffic light periods Δ_c is used as control signal and determined by the optimal strategy. The state equation of traffic system is as follows :

$$m[p](\tau) = m[p](0) + C.f[t](0).\Delta_c \qquad (4)$$

In the intersection scenario in Fig.7 and Fig.6 for the *Petri net* structure, Δ_c represent the firing time of the transition t_{19} as α (consist of t_7, t_9, and t_{11}) and t_{21} as β (consist of t_{29} and t_{33}). Both firing time have the value λ_{19} and λ_{21}, respectively. The type of transition t_{19} and t_{21} are discrete deterministic *Petri net*. The value of λ_{19} sets the time duration of horizontal traffic flow, joining S1 and S3 and λ_{19} sets the time duration of vertical traffic flow, joining S2 and S4.

A parameter optimization problem is introduced and solved for this intersection model (see Fig. 6), obtaining thus the optimal green periods for the traffic light. Notice that, since the yellow periods are fixed a priori for safety reasons, defining the green periods for each queue the red ones are completely determined, obtaining thus the complete timing for the traffic light.

Optimal feedback controller is used to obtain the optimal Δ_c (α and β) by minimizing the objective function (T_{delay}) within the range of maximum and minimum limit possible values of the traffic light period [3], Fig. 9 for illustration.

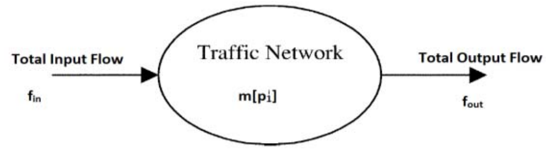

Fig. 9. Traffic Network Illustrator

$$T_{delay} = Num\ of\ vehicles\ in\ the\ road + Num\ of\ incoming\ vehicles - Num\ of\ out\ vehicles$$

So, the equation of objective function is

$$T_{delay} = \sum_{p_1^i} \int_0^{\rho} m[p_1^i](0)d\tau + \sum_{t\epsilon T_{in}} \int_0^{\rho}\int_0^{\tau} f[t](\xi)\, d\xi d\tau - \sum_{t\epsilon T_{out}} \int_0^{\rho}\int_0^{\tau} f[t](\xi)\, d\xi d\tau \qquad (5)$$

where $m[p_1^i]$is number of vehicles in the road during ρ time units (1 cycle).

VI. Simulation and Analysis

The intersection model for this scenario consists of four road section arm. Each road arm has two way traffic flow and two traffic lights to regulate the straight flow movements and turn right flow

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movement. A total of eight traffic lights are used to control the trafic flow in this intersection.

Parameters, used in this simulation, are created based on the data traffic counting. The data for each road section arm data consist of three traffic flows capacity, which are straight-, turn right- and turn left flow capacity. The total capacity S1 for straight-, turn right- and turn left flow are 47 vehicles, 15 vehicles, and 10 vehicles, respectively. The total capacity S2 for straight-, turn right-, and turn left flow are 20 vehicles, 30 vehicles, and 10 vehicles. The total capacity S3 for straight-, turn right, and turn left flow are 40 vehicles, 20 vehicles, and 10 vehicles. The total capacity S4 for straight-, turn right- and turn left flow are 30 vehicles, 20 vehicles, and 10 vehicles. The arc weights of *Place* for the road (S1, S2, S3, S4) to its *Transition* are q=100 and r=80. The initial values of *Places* are $m[p_1] = 20,\ m[p_7] = 11,\ m[p_{23}] = 18,\ m[p_{29}] = 20, m[p_{47}] = 3,\ m[p_{53}] = 26,\ m[p_{69}] = 18,$ $m[p_{75}] = 10$. The initial value of *Places* limited by road section capacity are

Because the route's capacity is limited, then the initial value for place $p_5, p_{11}, p_{27}, p_{33}, p_{51}, p_{57}, p_{73}, p_{79}$ are limited to $m[p_5]$=27, $m[p_{11}]$=4, $m[p_{27}]$=2, $m[p_{33}]$=20, $m[p_{51}]$=17, $m[p_{57}]$=4, $m[p_{73}]$=2, $m[p_{79}]$=20, Period time of dicretization are 2 seconds, $\Delta = 2$. The control period for one intersection traffic light cycle is 220 seconds. Th the yellow traffic light time period is 2 seconds.

A. Fix Control

The control system are simulated in this research with a ratio $\alpha = 80$ and $\beta = 30$, which mean the time taken by horizontal traffic flow for S1 - S3 is 160 second and time taken by vertical traffic flow for S2 and S4 route is 60 seconds (see Fig. 10 and Fig. 11). This ratio was taken based on the historical data of the flow of incoming vehicles in each lane.

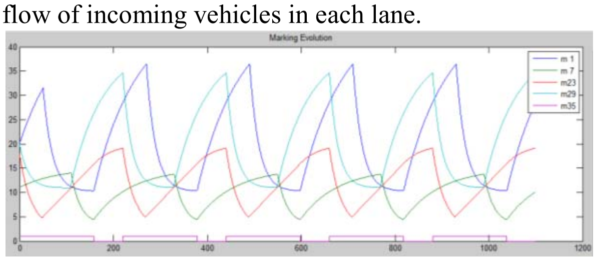

Fig. 10. Density in straight way and turning right in intersection (S1and S3) with *Fix Control*

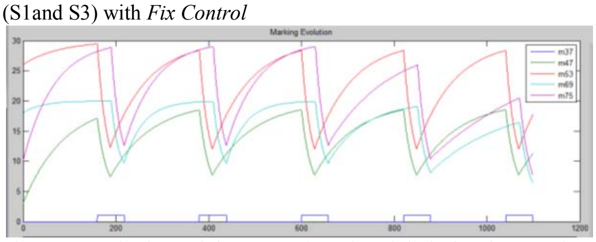

Fig..11. Density in straight way and turning right before intersection (S2and S4) with *Fix Control.*

The simulation during the red light for straight flow shows, that are 32-37 vehicles are queueing from S1 to S3 and 35 vehicles are queueing from S3 to S1. Meanwhile, during the red light for the turn right flow shows, that are 14-15 vehicles are queueing from S1 and 19-20 vehicles are queueing from S3. The queueing result of 20 vehicles for S3 and 15 vehicles for S1 shows, that there are unbalance traffic density in turn right flow.

B. Optimal Control

The optimal control system simulated in this research with a ratio $\alpha = 55$ and $\beta = 55$, which mean that the time taken by horizontal traffic flow for S1 - S3 is 110 second and time taken by vertical traffic flow for S2 and S4 route is 110 seconds (see Fig.12 and Fig.13).

In this simulation during the 4th cycle, the inflow S4 will be reduce to half, sothat the ratio are $\alpha = 65$ and $\beta = 45$. This ratio was taken based on the recent condition of inflow in each lane.

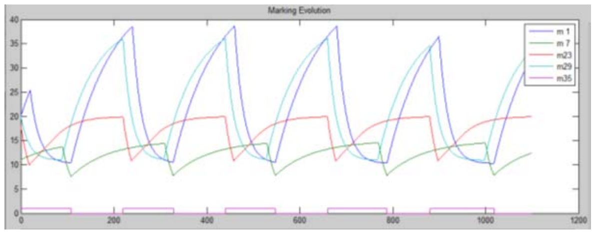

Fig. 12. Density in straight way and turning right in intersection (S1and S3) with *optimal control*

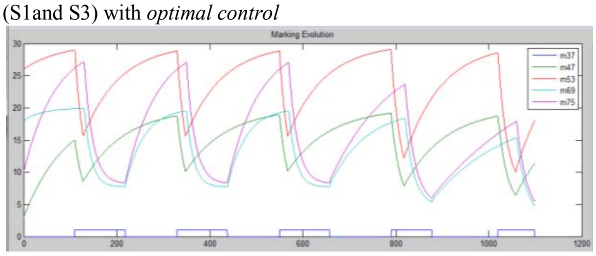

Fig..13. Density in straight way and turning right before intersection (S2and S4) with *optimal control.*

The result of simulation during the red light for straight flow shows, that are 25-39 vehicles are queueing from S1 to S3 and 36 vehicles are queueing from S3 to S1. Meanwhile, during the red light for the turn right flow shows, that are 14-15 vehicles are queueing from S1 and 20 vehicles are queueing from S3. The queueing result of 20 vehicles for S3 and 15 vehicles for S1 shows, that there are unbalance traffic density at the beginning in turn right flow. These unbalance of traffic are optimized as shown in Fig. 13. The optimal control rule was computed and applied each 220 time units (the computation of the control law takes 14.556 seconds was run under Matlab 7.8.0 on a CPU with Intel Core i5 at 2.27GHz).

C. Analysis

The traffic lights setting with fixed control in the simulation is done with a time duration of 220 seconds per cycle. This time duration consist of 160 seconds for a total time of horizontal flow (i.e: 110 seconds for a straight flow and 50 seconds for a turn right flow) and 60 seconds for a total time of vertical flow (i.e.: 30 seconds for S2 section and 30 seconds for S4 section).

As can be seen in the simulation results in section VI-A for a fixed control setting for the turn right flow cycle, the remaining number of vehicles during the green light for S3 are 5 vehicles and for S1 are 4 vehicles. However, the remaining number of vehicles during the red light for S4 indicate a long queues and jammed. This occured, because the input stream S3 is high at 0.3 - 0.4, but it has too short time for the green light (i.e. 30 seconds). This indicates the balance does not occur in the determination of traffic lights. It leads to congestion in a particular lane.

TABLE I
NUMBER OF VEHICLES FOR FIX AND OPTIMAL CONTROL

Cycle	Route (on the green light condition)	Num of Vehicles	
		Fix Control	Optimal Control
1	S1 Straight Way	32-11	25-10
	S1 Turning Right Way	14-4	14-8
	S3 Straight Way	20-11	20-11
	S3 Turning Right Way	18-5	18-10
	S2 Straight Way	17-8	15-8
	S2 Turning Right Way	30-13	29-16
	S4 Straight Way	29-14	27-9
	S4 Turning Right Way	20-10	20-8
2	S1 Straight Way	36-11	39-10
	S1 Turning Right Way	14-4	14-8
	S3 Straight Way	35-11	36-11
	S3 Turning Right Way	19-5	20-10
	S2 Straight Way	18-8	18-10
	S2 Turning Right Way	28-13	29-16
	S4 Straight Way	29-14	27-9
	S4 Turning Right Way	20-10	20-8
3	S1 Straight Way	36-11	39-10
	S1 Turning Right Way	14-4	14-8
	S3 Straight Way	35-11	36-11
	S3 Turning Right Way	19-5	20-10
	S2 Straight Way	18-8	18-10
	S2 Turning Right Way	28-13	29-16
	S4 Straight Way	29-14	27-9
	S4 Turning Right Way	20-10	20-8
4	S1 Straight Way	36-11	39-10
	S1 Turning Right Way	14-4	14-8
	S3 Straight Way	35-11	36-11
	S3 Turning Right Way	19-5	20-10
	S2 Straight Way	18-8	18-8
	S2 Turning Right Way	28-13	29-13
	S4 Straight Way	25-10	22-6
	S4 Turning Right Way	18-8	18-5
5	S1 Straight Way	36-11	35-10
	S1 Turning Right Way	14-4	14-8
	S3 Straight Way	35-11	34-11
	S3 Turning Right Way	19-5	20-10
	S2 Straight Way	18-8	18-6
	S2 Turning Right Way	28-13	29-10
	S4 Straight Way	20-8	27-6
	S4 Turning Right Way	15-7	14-5

Traffic lights setting to control optimization in the simulation to obtain optimal results based control strategy is the duration per cycle for 220 seconds in which to horizontal area total time of 110 seconds (for a straight lane for 90 seconds and a turn right lane 20 seconds) and for the area vertical of 110 seconds (for a straight lane for 20 seconds and a turn right lane 90 seconds). In 4^{th} cycle, to horizontal area total time of 130 seconds (for a straight lane for 110 seconds and a turn right lane 20 seconds) and for the area vertical of 90 seconds (for a straight lane for 30 seconds and a turn right lane 60 seconds). As can be seen in the simulation results in section VI-B, with optimal setting control the remaining number of vehicles visible when the green light in the turn right lane of S3 for 10 vehicles and in the turn right lane of S1 for 8 vehicles. The number of vehicles when the light is red in the turn right lane of S4 looks already more tenuous and not be longer queue than fixed control, because the timing to the green light in S4 section is enlarged. Although reducing the length of the green light on the other line, but it makes the road conditions become more tenuous and there is not congestion in a particular lane. Therefore, it is most optimal conditions for the flow of incoming vehicles to track S4 are greater than the turn right lane of S1 and S3.

VII. CONCLUSION

The two-ways model of an intersection with the 5 cycle phases based on a continuous *petri net* and controlled by 8 traffic lights based on discrete *petri net* has been modeled. Parameters used for the model are based on Pejaten Village's, South Jakarta intersection. The parameters are obtained by a traffic counting.

An offline optimal control has been applied to this model. The results of simulation using optimal control are better than the fixed control based on the density distribution at each road section arms, so that there is no congestion stuck at one lane. The computing time for an optimal control are faster than a cycle time of traffic light in this intersection.

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