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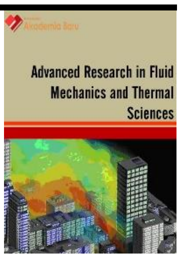

Stress Analysis of Thick-Walled Cylinder for Rocket Motor Case under Internal Pressure

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ABSTRACT

This paper investigated the stresses developed in a thick-walled cylinder for rocket motor case under internal pressure. Stress analysis used the finite element method with ANSYS software for rocket motor case selection. The simulation used thick-walled cylinders with wall thickness 6, 7, 8, 9, and 10 mm having 300 mm long and outer diameter of 122 mm. It used an internal pressure of 2, 4, 6, 8, and 10 MPa. The material variation in this research used Aluminium 6061, CFRP, and GFRP. Comparing the hoop and longitudinal stress values between analytics and simulations were for the validation process. The simulation results show that the thicker the cylinder wall, the von Mises stress decreases. Aluminium 6061 and CFRP have a safety factor greater than 1 for all wall thickness and internal pressure variations. GFRP has a safety factor greater than 1 for all internal pressure variations when the wall thickness of 10 mm.

1. Introduction

The rocket motor is an essential part of a rocket that contains fuels used to boost a missile [1]. It consists of a case, nozzle, cap, insulator, igniter, etc. It works using principles such as pressure vessels because it stores propellant fuel. Generally, rocket motor works in environments with high pressure and temperature [2].

The design of a rocket motor case depends on the internal pressure and material used. The rocket motor case that works at high internal pressure generally uses a thick-walled cylinder, while at low internal pressure uses a thin-walled. Materials have high yield strength typically use a thin-walled cylinder, while materials have low yield strength use a thick-walled.

The design of the rocket motor case uses a cylindrical theory to calculate the effect of the thickness of the case on the von Mises stress under internal pressure. Stress analysis used the finite

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element method with ANSYS software for rocket motor case selection. The simulation used thick-walled cylinders with wall thickness 6, 7, 8, 9, and 10 mm having 300 mm long and outer diameter of 122 mm. It used an internal pressure of 2, 4, 6, 8, and 10 MPa.

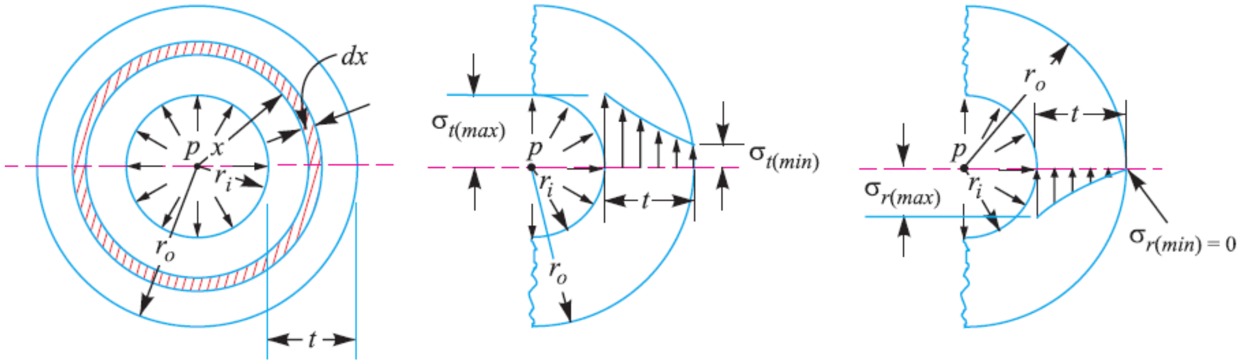
The finite element analysis is numerical mathematical techniques for calculating the strength and structural behavior of engineering components with dividing objects into mesh shapes. This simulation technique allows every design and product to be analyzed in great detail. It is widely used to measure stress and stress concentration in a thick-walled and thin-walled cylinder [3].

The goals of this investigation are to predict hoop and longitudinal stresses in thick-walled cylinders, von Mises stress distribution over the thickness of the thick-walled cylinder with closed ends, and select of materials which suite to these cases.

2. Stress in Thick-Walled Cylinder

The thin-walled and thick-walled are classifications of pressure vessels based on their dimensions. The thin-walled pressure vessel in which the wall thickness (t) is less than 1/20 of the internal diameter (d). On the other hand, the thick-walled in which the wall thickness (t) is more significant than 1/20 of it (d).

The rocket motor case in this study is assumed to be a thick-walled cylinder. In the case of a thick-walled, as shown in Figure 1(a), stress on parts of the wall cannot be assumed to be evenly distributed [4]. The tangential and radial stresses develop by considering values that depend on the radius of the element. Figure 1(b) and (c) show the stress distributions on thick-walled cylinders.



(a) (b) (c)

Fig. 1. Stress distribution in thick-walled cylinder subjected to internal pressure [4]. (a) Thick-walled cylinder (b)Tangential stress distribution (c) Radial stress distribution

Theory of Elasticity is the primary theory for the thick-walled cylinder, which produces a stress condition as a continuous function of the radius above the wall of the pressure vessel [5]. The stress in a cylindrical pressure vessel depends on the ratio of the inner radius to the outer radius (r_o/r_i) rather than cylinder size.

The thick-walled cylinder pressure will occur in three directions, namely hoop (circumferential or tangential), longitudinal (axial), and radial. The maximum stress equations in thick-walled cylinders in the hoop, longitudinal, and radial directions are

$$(\sigma_h)_{max} = p_i \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \tag{1}$$

$$(\sigma_l)_{max} = p_i \left(\frac{r_i^2}{r_o^2 - r_i^2}\right) \tag{2}$$

$$(\sigma_r)_{max} = -\, p_i \tag{3}$$

where p_i = internal pressure, r_o = outer radius of cylinder, and r_i = inner radius of cylinder

3. Finite Element Analysis

Figure 2 shows the model geometry of the cylinder. Since the geometry exhibits asymmetry in the axial (Y), then it can model only the top half of it [6]. The finite element analysis used a 90-degree segment of the solid model of the cylinder. The symmetric nature of the geometry and loading means that displacements are zero in directions normal to the face exposed by the vertical and horizontal cuts employed create this one-eight segment of the cylinder (Figure 3) [7].

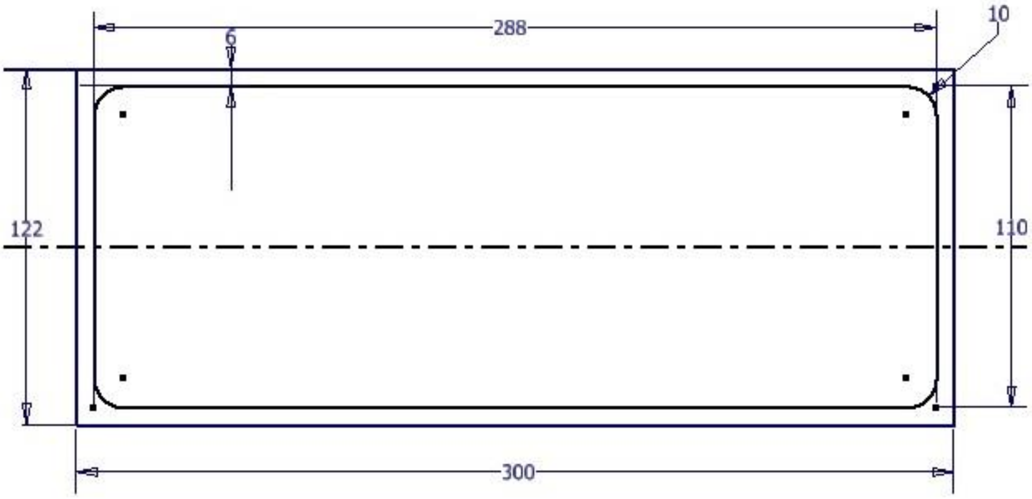


Fig. 2. Design of thick-walled cylinder with a wall thickness of 6 mm

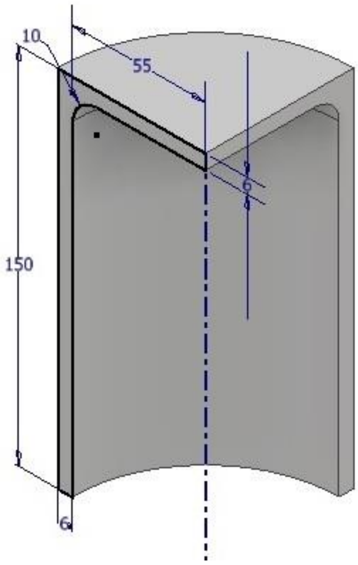


Fig. 3. The one-eight segment of the cylinder with a wall thickness of 6 mm

The following assumptions are made theoretically on thick-walled cylinders

1. Aluminium 6061 is homogeneous and isotropic. CFRP and GFRP are linearly elastic, initially free of stress and defect. The fiber and matrix are homogenous, linearly elastic, and isotropic.
2. The cylinder section remains flat even after applying internal pressure.
3. All cylinder surface will expand or contract independently.

The boundary conditions of the finite element method using ANSYS software are shown in Figure 4 and input parameters in Table 1.

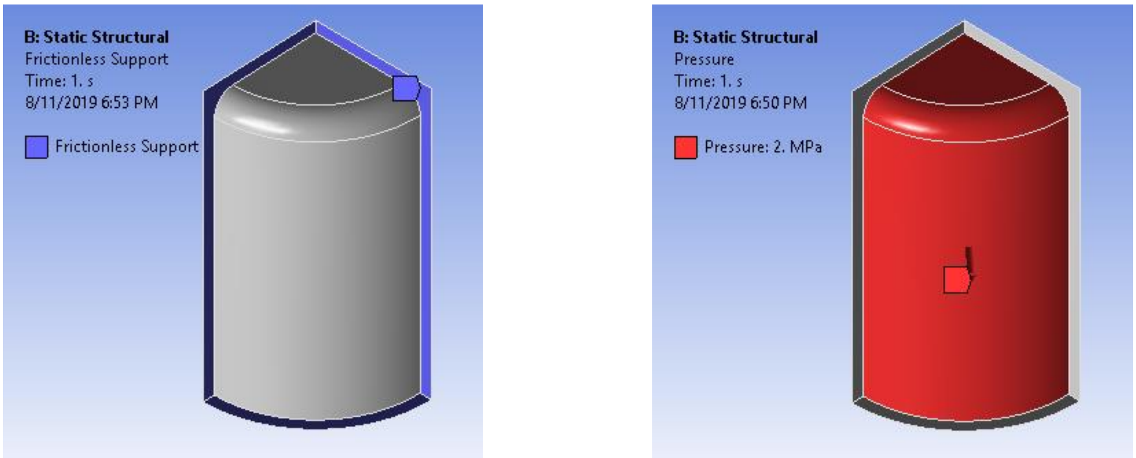


Fig. 4. The boundary conditions of the finite element method using ANSYS software

Table 1
Input parameters

Parameters	Symbol	Value
Length of cylinder	L	300 mm
Outer diameter of cylinder	D_o	122 mm
Wall thickness	t	6, 7, 8, 9, and 10 mm
Inner radius of cylinder	r_i	55, 54, 53, 52, and 51 mm
Internal pressure	P_i	2, 4, 6, 8, and 10 MPa
Element size	-	3 mm
Number of nodes	-	25194, 23872, 27096, 26341, 26926
Number of elements	-	13867, 13041, 15321, 14952, 15343

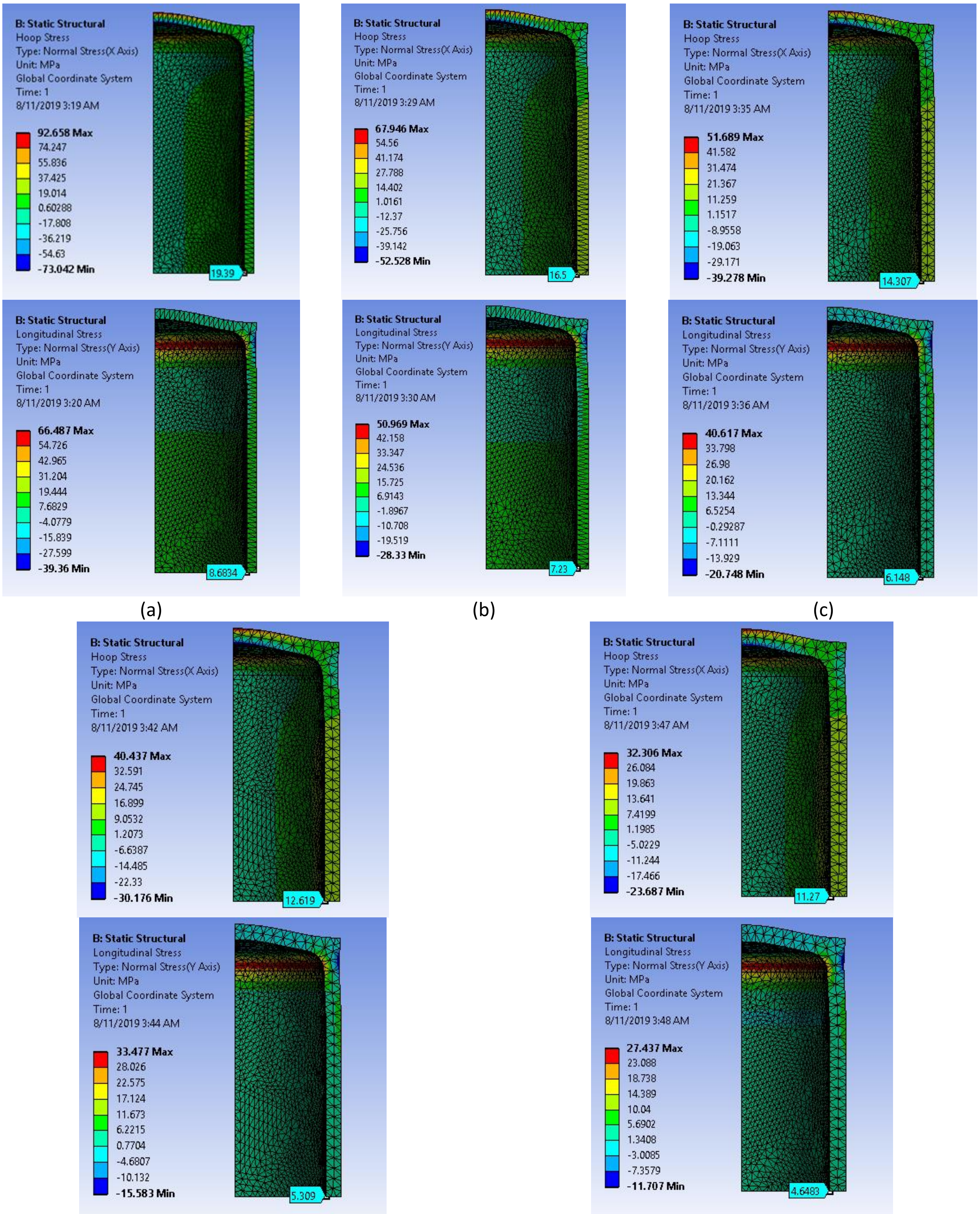
The criteria for rocket motor case material are lightweight and high strength. Aluminium 6061, Carbon Fiber Reinforced Polymer (CFRP), and Glass Fiber Reinforced Polymer (GFRP) are chosen for it [8-11]. The material properties of Aluminium 6061, CFRP, and GFRP are shown in Table 2. The data of mechanical properties of materials using Autodesk Material Library [12].

Table 2
Mechanical properties of materials

Material	Density (gr/cm^3)	Young modulus (GPa)	Tensile Strength (MPa)
Aluminium 6061	2.70	68.9	400
CFRP	1.43	133	577
GFRP	1.75	13.9	194

4. Result and Discussion

Figure 5 shows the effect of wall thickness on the hoop and longitudinal stress of the cylinder with an internal pressure of 2 MPa. The simulation results show that the thicker the cylinder wall, the maximum hoop and longitudinal stress decreases. The relationship between wall thickness and maximum hoop and longitudinal stress is inversely proportional.



(a) (b) (c)

(d) (e)

Fig. 5. Hoop and longitudinal stress of cylinder with wall thickness difference (a) 6 mm, (b) 7 mm, (c) 8 mm, (d) 9 mm, and (e) 10 mm with internal pressure of 2 MPa

Otherwise, if the internal pressure of the cylinder increases, then the maximum hoop and longitudinal stress increases (Table 3). The relationship between internal pressure and maximum hoop and longitudinal stress is linear. Both values of stress using finite element analysis are not significant difference using analytic — the error percentage between finite element analysis and analytical less than 1 percent.

The hoop stress is always tensile, and maximum hoop stress always occurs at the inner radius or the outer radius depending on the direction of the pressure gradient [13]. If the thick-walled cylinder subjected to external pressure only, then the maximum hoop stress occurs at the outer radius ($r=r_o$). In the research, the thick-walled cylinder subjected to internal pressure only, then the maximum hoop stress occurs at the inner radius ($r=r_i$).

When the cylinder has closed ends the internal pressure acts on them to develop a force along the axis of the cylinder. It is called the longitudinal stress and is usually less than the hoop stress. In the research, the hoop stress higher than longitudinal stress. The longitudinal stress values are about 41-44 percent than the hoop stress.

Aluminium 6061, Carbon Fiber Reinforced Polymer (CFRP), and Glass Fiber Reinforced Polymer (GFRP) can withstand maximum hoop and longitudinal stress. The three materials have a tensile strength exceeding the two maximum stresses (hoop and longitudinal). It means the materials can be used as rocket material case without end caps.

Table 3
Comparison between analytic and simulation of the maximum hoop and longitudinal stress with wall thickness and internal pressure variations

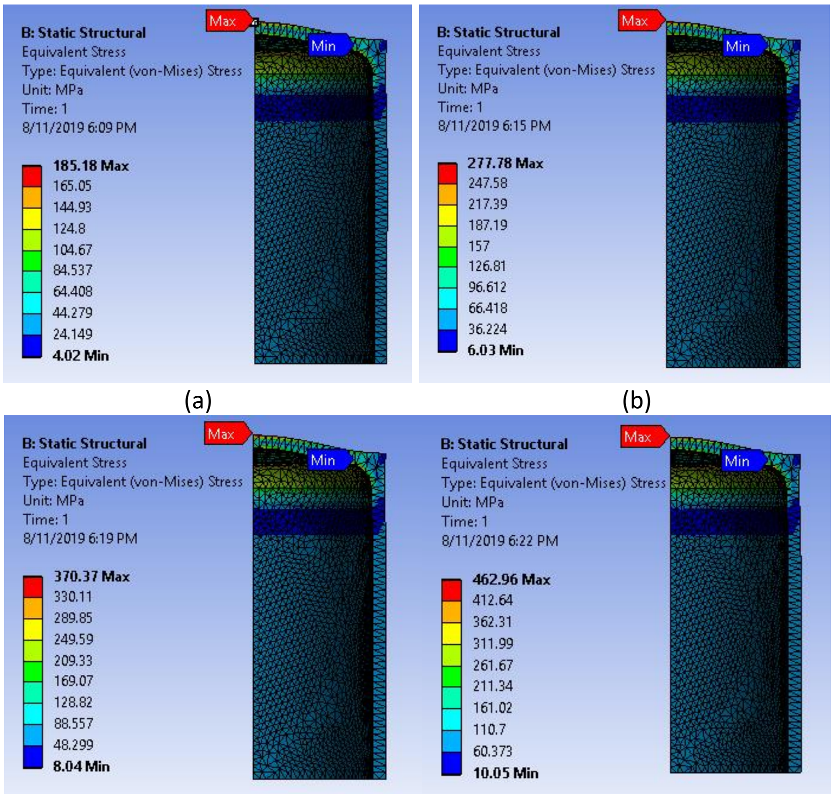
Thickness (mm)	Pressure (MPa)	Analytic		FEA		Error	
		Hoop stress (MPa)	Long. Stress (MPa)	Hoop stress (MPa)	Long. Stress (MPa)	Hoop stress (%)	Long. Stress (%)
6	2	19.39	8.69	19.39	8.68	0.00	0.12
7	2	16.49	7.24	16.5	7.23	0.06	0.14
8	2	14.32	6.16	14.31	6.15	0.07	0.16
9	2	12.64	5.32	12.62	5.31	0.16	0.19
10	2	11.29	4.64	11.27	4.65	0.18	0.22
6	4	38.77	17.37	38.77	17.37	0.00	0.00
7	4	32.98	14.49	32.99	14.46	0.03	0.21
8	4	28.64	12.32	28.62	12.3	0.07	0.16
9	4	25.27	10.64	25.24	10.62	0.12	0.19
10	4	22.58	9.29	22.55	9.30	0.13	0.11
6	6	58.16	26.08	58.16	26.05	0.00	0.12
7	6	49.47	21.73	49.49	21.69	0.04	0.18
8	6	42.96	18.48	42.92	18.44	0.09	0.22
9	6	37.91	15.95	37.86	15.93	0.13	0.13
10	6	33.87	13.93	33.82	13.95	0.15	0.14
6	8	77.54	34.77	77.54	34.73	0.00	0.12
7	8	65.96	28.98	65.99	28.92	0.05	0.21
8	8	57.28	24.64	57.23	24.59	0.09	0.20
9	8	50.54	21.27	50.48	21.24	0.12	0.14
10	8	45.16	18.58	45.1	18.59	0.13	0.05
6	10	96.93	43.46	96.93	43.42	0.00	0.09
7	10	82.45	36.22	82.49	36.15	0.05	0.19
8	10	71.6	30.8	71.54	30.74	0.08	0.19
9	10	63.18	26.59	63.1	26.55	0.13	0.15
10	10	56.45	23.22	56.37	23.24	0.14	0.09

One of the theories of failure is based on maximum distortion energy, which is known as the von Mises yield criterion. The von Mises stress is also known as the equivalent stress. The von Mises stress counts what combination of stress at a certain point will cause failure. In other words, it says that the material will fail when the equivalent stress exceeds the yield strength of the material [14,15].

Figure 6 shows the maximum von Mises stress of cylinder for wall thickness of 6 mm with an internal pressure difference. The simulation results show that the thicker the cylinder wall, the von Mises stress decreases (Table 4).

The relationship between wall thickness and von Mises stress is inversely proportional. Otherwise, if the internal pressure of the cylinder increases, then the maximum von Mises stress increases (Table 4). The relationship between internal pressure and maximum von Mises stress is linear.

The maximum von Mises stress always occurs at the outer radius of the end cap. It means the outer radius of the end cap is a critical area. The maximum von Mises stress can be decreased with increasing the end cap thickness, although without increasing the wall thickness. In other words, the method is more efficient because the material is lighter than increase overall cylinder thickness. It can also be done by combining different materials. For example, the tube material is made of GFRP while the end cap material is made of Aluminum 6061.



(a) (b)

(c) (d)

Fig. 6. The maximum von Mises stress of cylinder for wall thickness of 6 mm with an internal pressure of (a) 4 MPa, (b) 6 MPa, (c) 8 MPa, and (d) 10 MPa

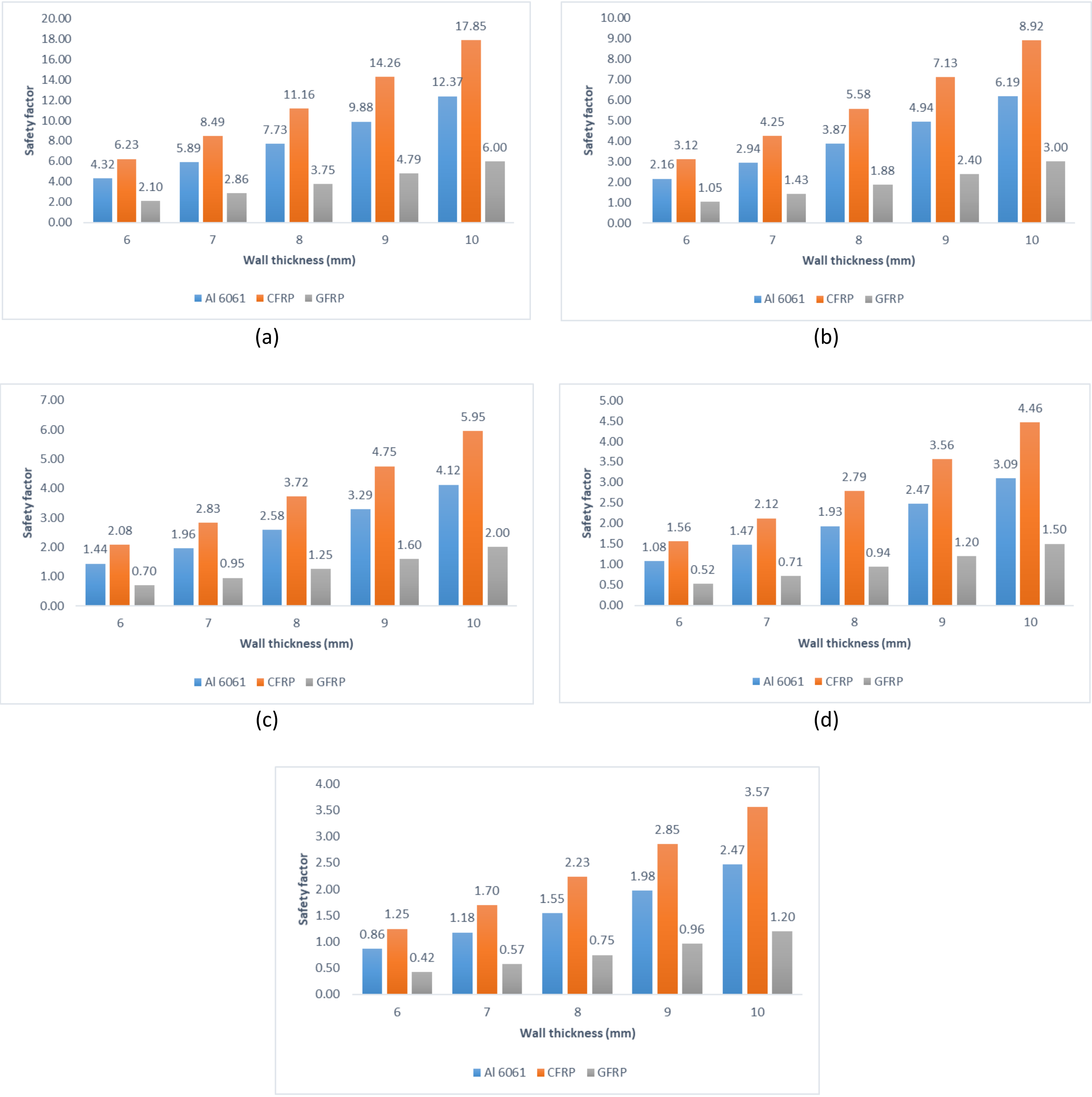
Table 4
The value of maximum von Mises stress

Thickness (mm)	Pressure (MPa)	Von Mises stress (MPa)
6	2	92.59
7	2	67.93
8	2	51.72
9	2	40.47
10	2	32.33
6	4	185.18
7	4	135.86
8	4	103.45
9	4	80.93
10	4	64.66
6	6	277.78
7	6	203.79
8	6	155.17
9	6	121.40
10	6	96.99
6	8	370.37
7	8	271.72
8	8	206.90
9	8	161.86
10	8	129.32
6	10	462.96
7	10	339.65
8	10	258.62
9	10	202.33
10	10	161.65

The safety factor is used to perform the evaluation process so that the components or structures design is guaranteed safe even though they used the minimum dimensions [16]. It is one indicator of the strength of an element. Parts are said to be safe when the maximum von Mises stress is lower than the strength of the material. In other words, the component is safe when the safety factor value is more than 1.

The value of the safety factor for each material shown in Figure 7. Aluminium 6061 and CFRP have a safety factor greater than 1 for all wall thickness and internal pressure variations.

GFRP has a safety factor greater than 1 when the wall thickness of 8 mm, 9 mm, and 10 mm at an internal pressure of 6 MPa. At the internal pressure of 8 MPa, GFRP has a safety factor greater than 1 when the wall thickness of 9 mm and 10 mm. At the internal pressure of 10 MPa, GFRP has a safety factor greater than 1 when the wall thickness of 10 mm. In other words, GFRP has a safety factor greater than 1 for all internal pressure variations when the wall thickness of 10 mm.



(a) (b)

(c) (d)

(e)

Fig. 7. Effect of the wall thickness on safety factor using different materials with internal pressure of (a) 2 MPa, (b) 4 MPa, (c) 6 MPa, (d) 8 MPa, and (e) 10 MPa

5. Conclusion

The simulation results show that the thicker the cylinder wall, the maximum hoop and longitudinal stress decreases. The validation of analytical results is done with finite element analysis for the maximum hoop and longitudinal stress with fixed boundary condition. The results showed the error percentage of less than 1 percent.

Von Mises is chosen as a failure criterion. The simulation results show that the thicker the cylinder wall, the von Mises stress decreases. Aluminium 6061 and CFRP have a safety factor greater than 1 for all wall thickness and internal pressure variations. GFRP has a safety factor greater than 1 for all internal pressure variations when the wall thickness of 10 mm.

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