DYNAMICAL ANALYSIS OF INTERNAL SOLITARY WAVES PROPAGATION OVER UNEVEN BOTTOM IN THE LOMBOK STRAIT

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Abstract.Internal solitary waves (ISW) often observed in the Lombok strait which is the main pathway of the Indonesian throughflow. ISW can be detected by Synthetic Aperture Radar (SAR) as a dark band followed immediately by a bright band of curvilinear wave crest due to the reverse of circulation flow induced by the waves. In this paper, dynamical analysis and the properties of ISW in the northern part of the Lombok strait are investigated by combinaton ALOS-PALSAR II observation and mathematical modeling. The SAR data observed on May22th 2016 and September, 19th 2016 is used in this study. Dynamical properties of ISW are studied by the variable Perturbed Kortweg and de Vries equation (vPKdV). The short derivation of vPKdV based on two layer fluid model will be given. The result showed that the basic forms of ISW observed is the arc-like ISW that radiates uniformly away in regular sequences from the sill in the north. We found that the main wave packet has the wavelengths is more than2000 m and the amplitudes is about 30m with the nonlinear phase speed is about 12 m/s.The changing of the wave form from depth water into shallow water and breaking criteria will be highlighted.

1. Introduction

Internal solitary waves (ISW) is one of the interested phenomena in the ocean. This is an nonlinear wave that travel within the interior of the water colomn. This waves are frequenctly observed all over the words ocean that ussually associated with the strong tides in the strait and also varying topography feature (Apel et al. 1995). In the interior of the ocean, ISW can be observed by the existence of a varying of temperature or salinity in water colomn. ISWs are important for many reasons due to they can propagate over several hundred kilometers and transport both energy and mass. In practical purpose the ISWs are influence on the oil-drilling rigs activity, the mariculture activity due to the ISW often induce mixing which is an important factor for modifying the biogical activity. The ISW also can generate the sediment resuspension where it is responsible for pollutant dispersion of the bottom of the coastal area (Alpers1985, Apel, Farmer and Smith 1999).

It is interesting that the existence of ISW can be detected by Radar Images espsecially Synthetic Aperture Radars (SAR). Due to the ISWs are coherence processes they will expresses in a specific pattern. In the Radar backscatter the ISW showed the light/dark signature. The dark signature is come from the very low rough surface and the light is come from the rough surface associated with the orbital current of ISW (the elevation). Generally the signature of the ISW in the SAR images is a lineal feature by a bright region, then a dark region and finally followed by normal roughness (Apel et al. 1995, Alpers1985). The other words, the ISW cause a pattern of converging (bright) and diverging (dark) current under the sea surface.

The one of area with ISW very active in the world is found in the Lombok strait. The strait, a small sea channel between the islands Bali and Lombok, is the most important the main pathway of the Indonesia Troughflow (IT). It transports a large amount of relative warm water from the Pacific

Ocean into the Indian Ocean annually (Gordon2005). ISW in the Lombok strait is generated by strong tidal flow of stratified water over shallow sills. Previous study showed that ISW in the strait propagate with velocity is about 1.8 m/s with the wavelengths are about 2-4 km, and propagate in three different pattern: southward, northward and both direction (Susanto et al. 2005). The ISW observed every month and occur more than 30 ISW exist every year (Karanget al. 2012). Numerical study by using nonhydrostatic model showed that ISWs are more numerous on the north path with 2.6 GW energy of the wave propagate to the Kangean island (Aiki et al. 2011). Observation by SAR on April, 13 2000 showed that in the north parth, ISW propagate with phase velocity is about 2.2 m/s and the amplitude 20m and associated orbital velocity is 0.14m/s (Sulaiman,and Sadly 2006). Another study showed that the ISW wavelength are about more than 2km and the amplitude is about 20m (Susanto et al. 2005). There are still unresolved issues related with the propagation of ISWs, in particular the dissipation processes in the north path of Lombok strait.

In this paper, ISW in the north part of the Lombok strait is investigated. Alos-Palsar II images and KdV model will be used to analysis the dynamics of ISW with taking into account the effect of topography. This study can answer the question about dissipation and the breaking of ISW in the north side of the Lombok strait. The paper is organized as follows, the data and methodology related to data processing and mathematical models are descried in Sec.2. The analysis and discussion wil be given in Sec 3. The paper will be ended by a conclusion.

2. Data and Methodology

2.1 Alor Palsar II Images

The Alos-Palsar II on on May22th 2016and September, 19th 2016 are used in this study. The data was provided by JAXA under Alos Plasar II research announcement project (PI-417002). SNAP-S1TBX (sentinel application platform-sentinel 1 toolbox is used to processed the data. The data processing step such as converting Alos-2 ceos data format to BEAM-DIMAP data format, radiometric calibration, reduce speckle noise by applying refined Lee filter and converting the intensity to decible were done. The extraction of ISW from SAR image is processed by using Matlab 7.7.

2.2 The Model for Internal Solitary Waves Propagation

The most of accepted model of ISW was described by Korteweg de Vries (KdV) equation. The equation describe a weakly nonlinear and a unidirectional waves propagation in one-dimensional (*x,t*). The equation showed that the ISW propagate as a balance between the nonlinear effect and the dispersion effect. The equation have been derived for both continus stratified or two layer ocean (Gerkema1994). If $\eta(x,t)$ is the amplitude of ISW then two layer approximation the KdV equation is given by (Grimshaw 1997),

$$\frac{\partial \eta}{\partial t} + c_0\left(\frac{\partial \eta}{\partial x} + \alpha\eta\frac{\partial \eta}{\partial x} + \gamma\frac{\partial^3 \eta}{\partial x^3}\right) = 0 \qquad (1)$$

with the coeffcient is,

$$c_0 = \sqrt{\frac{g(\rho_2-\rho_1)H_1H_2}{\rho_2H_1+\rho_1H_2}} \qquad \alpha = \frac{3}{2}\frac{c_0}{H_1H_2}\frac{\rho_2H_1^2-\rho_1H_2^2}{\rho_2H_1+\rho_1H_2} \qquad \gamma = \frac{c_0H_1H_2}{6}\frac{\rho_1H_1+\rho_2H_2}{\rho_2H_1+\rho_1H_2} \qquad (2)$$

where c_0 is long-wavelength phase speed, α and γ are nonlinear and dispersion coefficient respectively. From the SAR g is gravitational acceleration, ρ_1 and ρ_2 are the sea water density of the upper and lower layer respectively. $H_1$and H_2 are the depth of the upper and lower layer. The boundary between H_1 and H_2 is called pynocline line. The Eq. (1) describe unidirectional ISW propagate in the constant water depth. By assumming the dissipation and the breaking of ISW may be caused by topography effect then development of KdV equation in varying depth is needed. The varying depth should be appeared in the phase velocity term so that it can describe the breaking of ISW. The wave breaking indicate with increasing phase velocity in varying depth (Grimshaw 1997). The KdV equation with taking into account of topography is described by (Sulaiman2016):

$$\frac{\partial \eta}{\partial t} + c_0\left(\frac{\partial \eta}{\partial x} + \alpha\eta\frac{\partial \eta}{\partial x} + \gamma\frac{\partial^3 \eta}{\partial x^3}\right) - \delta\frac{\partial h}{\partial x}\eta = 0 \qquad (3)$$

where $\delta = (1/2c_0^2)(H_1/H)^2/((H_1/H-1)H - H_1/H)^2$, $h = H_1 + H_2(x)$
and H is the average depth. This is called the variable-coefficient perturbed Korteweg and de-Vries (vP-KdV) equation.

3. Results and Discussion

The existence of ISW observed by using ALOS 2 is depicted in Fig-1.

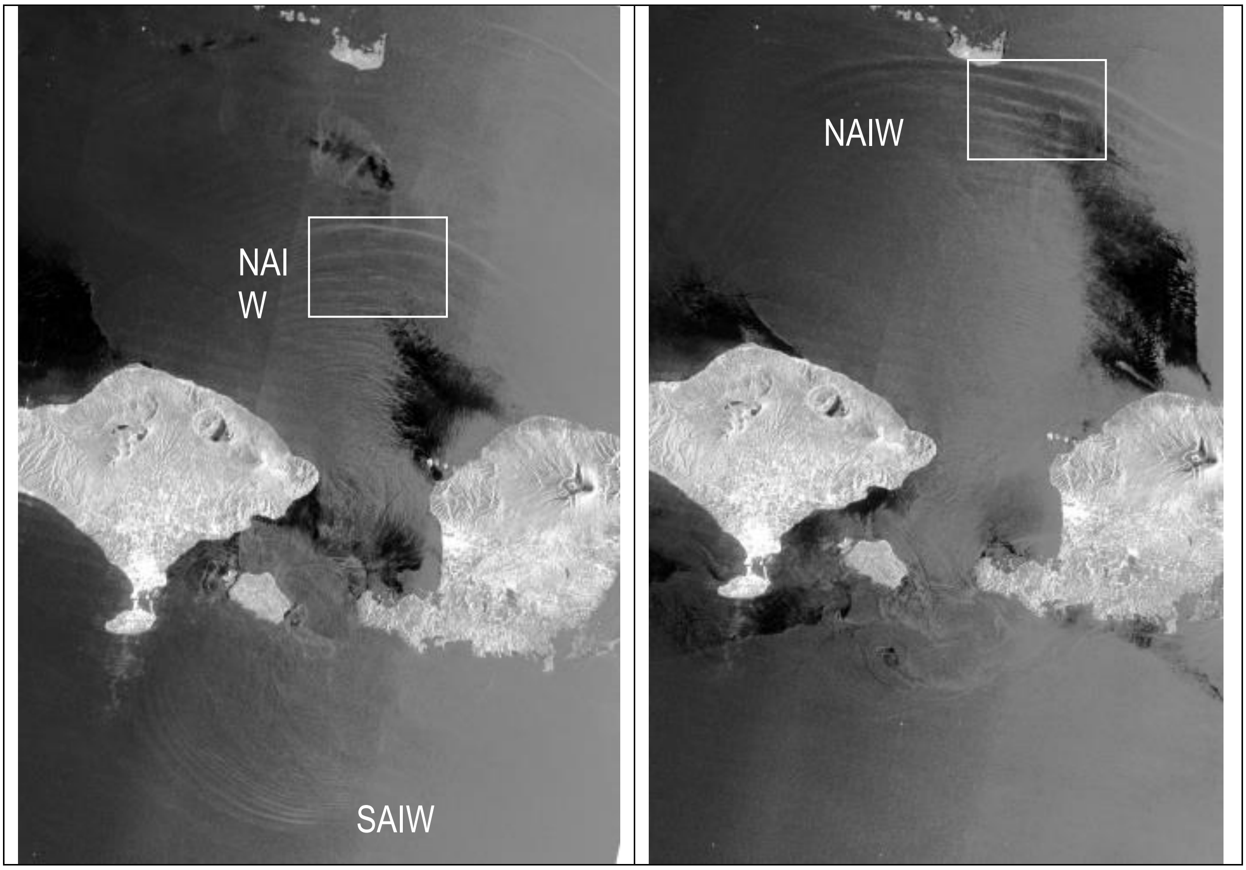

Figure-1. The Alos-Palsar images of the existence of ISW in the Lombok strait. a) May, 22 2016 and b) September, 19 2016.

From the figure show that they are two type of ISW, the arc-like internal wave (AIW) that propagate to the North here we denote NAIW. The second is arc-like internal wave that propagate in the South (SAIW) that exist in the south part. From the data ERS1/2 ranging from 1996 to 2001 suggests that the ISW were observed many times but not every month (Aikiet al. 2011, Matthewset al. 2012). The data also report that NAIW and SAIW appear at the same time i.e. at the north west monsoon (NWM). Further, based on ALOS image from 2006 to 2011 show that internal wave observed throughout the year. Most appearances ussualy at south east monsoon (SEM) and NWM but weakened during the transitional monsoon(Karanget al. 2012). The figure observed at transitional monsoon so that we get the weaken ISW. On september, the SAIW is not observed but the big NAIW propagate to the Kangean island. Soliton tend to occur in wave packets which is ussually in rank-ordered with the largest oscillation appearing at the packet front. But in the Nort path we found the ISW is not in the packet form but in “the periodic” forms.

In the paper we consider the propagation of the NAIW with taking into account of topography effect. First we study the wavelenght of ISW. The wavelength can be obtained by slection of specific region and then plot the backscatter intensity with pixel value. The wavelenght is associated by the distance of the bright (high) and the dark (low) backscatter intensity. The ISW wavelength determined by empirical formula D=1.32L where D is the distance between dark and bright of SAR images and L is the internal solitary wave half width(Sulaiman2016). The extraction of ISW from ALOS image is depicted in Fig.2.

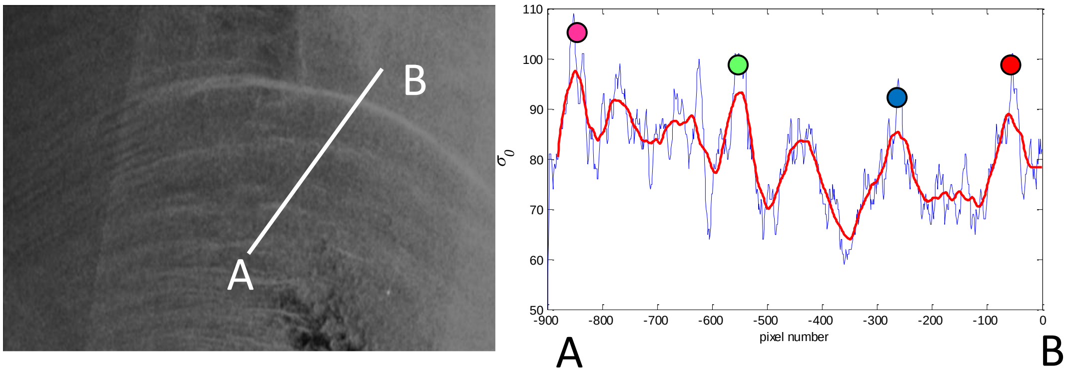

Figure 2. a) NAIW patternin Fig 1.a, obtained by applying the lowpass. The cycle is the high backscatter value.

From the Fig.2, we have four high backscatter value that accosicated with the amplitude of ISWs. The first soliton (from right B to left A) we have D=90 pixel number = 2250m. This mean that the ISW wavelenght is about $\lambda=2L=2D/1.32$= 3787.9 m. The topography in this region relatively flat so that the ISW propagation can be obtained by applying the soliton solution of Eq.(1) as follow (Apel 1995, Susanto et al. 2005, Sulaiman and Sadly 2006):

$$\eta = 2\eta_0 \sec h^2\left[\frac{(x-Vt)}{\Delta}\right] \qquad (4)$$

where $V=c_0(1+2/3\ \alpha\eta_0)$ is the nonlinear phase speed and $\Delta=\sqrt{(6\beta/\alpha\eta_0)}$ is a measure of the width of the squared hyperbolic secant pulse. This is related to the wavelength is $\lambda=2\Delta$. Another solution for describe the periodic soliton is given interm of cnoidal wave (cn_s) as (Apel 1995),where s is a modulus. When $s=1$ is become $cn \sim sech$ and this is nothing else a single pulse soliton. The ISW profile estimate based on the Fig.2 is depicted in Fig.3.

The figure show that the first soliton propagate with the speed is about 11.4 m/s and the wavelength 3409.1m, the second, the third and the forth ISW propagate with the velocity 11.5 m/s, 12,9 m/s and 14.6 m/s respectively and the wavelength is about 3302.3m, 3219.8 m and 3030.3m respectively. The wavelength is depending on the amplitude and the speed. If the wavelength longer then the amplitude will increase and then followed by the the increase of the velocity. Ussually the ISW form will be changed by the topography effect but in this case the topography relatively flat. It is mean that there are another mechanism of the damping of ISWs. It is speculatted that the damping may be caused by the wave-current interaction due to the present of strong tidal current (3.5 m/s) in the Lombok strait. The previous study by using ESR1/2 sattelite confirm that the wavelengths are about 2-7 km and the amplitude in order 20m (Susanto et al. 2005, Matthews et al. 2012, Mitniket al. 2000).

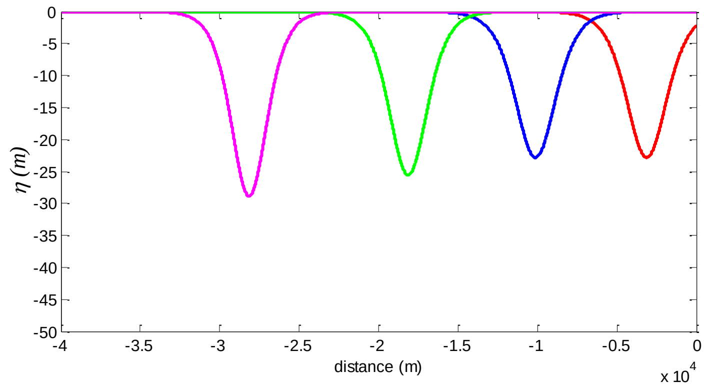

Figure 3. ISW profile estimation from SAR images of Fig 2. The red, blue, green and magenta color is the ISW profile associated with the bright signyal in Fig2b. The wave propagate from left to right. The environmental effect such as density and topography were obtained by using NODC global oceanographic data. The η=0 is the pynocline layer.

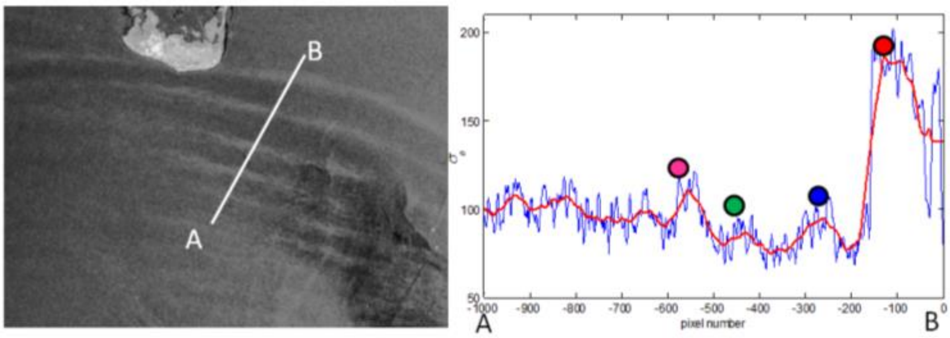

Figure 4. The ISW pattern obtained from Fig 1b) where the cycle associated with the bright radar backsactter.

The ISW profile estimation based on the Fig 4 is depicted in Fig 5. At the depth 250m with the pynocline layer is about 100m the ISW have a different form. The displacement is upper from the pynocline layer. This is similar as a surface wave where the atmosphere layer replace by H_1 layer.

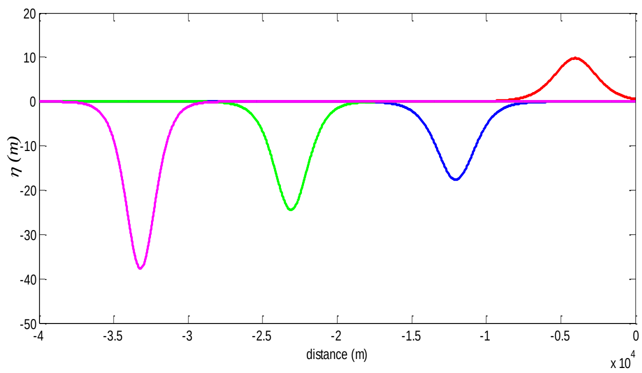

Figure 5 The ISW profile estimated based on Fig 4 with single soliton solution. The color is associated with the color in Fig 4.

The first soliton at the depth of 300m have the wavelength 3787.9 m and the phase speed 0.8 m/s. The wavelength at the depth of 500m, 700m and 1000m have the wavelength are 3409 m, 3030m and 2650 m respectively and the phase speed are 4.5m/s, 10m/s and 14.6m/s. The decreasing of the water depth will increase of the wavelength and decrease the phase speed. When the depth go to shallow then the amplitude will decrese and finally disappear.

The result above is obtaned by using single soliton solution Eq.4. This solution can not explain the mechanism of the changing of ISW due to topography effect. The effect of topography to ISW propagation is described by Eq. 3. The single solution based on the weakly nonlinear approximation is described by (Grimshaw 1997, Sulaiman2016),

$$\phi(\varsigma,\tau) = \phi_0 \operatorname{sec}h^2\left[\gamma\left(\varsigma - \overline{V}\tau\right)\right] + \varepsilon\phi_1 \operatorname{sec}h^2\left[\sqrt{\frac{A_1}{2}}\left(\varsigma - \overline{V}_1\tau\right)\right] \qquad (4)$$

Where ε is a small parameter, $\phi_0=3V/\delta_1$, $\gamma=\sqrt{(\delta_1\phi_0/12\delta_2)}$, $\phi_1=\sqrt{(V_1^2/4+2\phi_0+\phi_0^2)}-V^2/2$. We use the coordinate transfor as follow,

$$\phi = \exp\left[-\int \delta \frac{\partial h}{\partial x'} dx'\right]\eta \quad ; \quad \tau = \int^{s} ds/c \quad ; \quad \varsigma = \tau - x \quad ; \quad \delta_1 = \frac{3}{2ch} e^{-\int \delta \frac{\partial h}{\partial x'} dx'} \quad ,(5) \delta_2 = \frac{h^2}{6c^2}$$

In principle, by specify the topography form such as *h(x)= tanh(x)* then the solution represent the propagation of ISW with varying depth. This solution is only valid for assumption that the topography vary slowly. This mean that the topography undulation is more larger than ISW wavelength (Apel2003). The application of this solution need a pricise data of topography and environmental condition such as varying salinity, temperature and tidal current. This work is still in progress.

4. Conclusion

The existence of ISW in the lombok strait based on ALOS II images have been investigated. There are two tyoe of ISW i.e. the NAIW in the north side of the strait and the SAIW in the south path. Based on the single soliton solution we show that there are four main ISWwith the wavelength is about 2-4 km and the phase speed is about 10m/s for NAIW. The amplitude of ISW is about 25m. The behavior of ISW due to varying topography showed that the ISW wavelenght will be longer when the topography go to shallow.

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