NORMAL MODE ANALYSIS OF N219 WING FOR B-11 CONFIGURATION

Kosim Abdurohman[1)], R.M. Bayu Sakti[2)], Patriot Wismoyo Parlindungan Silitonga[2)]
National Institute of Aeronautics and Space Indonesia (LAPAN) [1)]
Indonesian Aerospace (IAe) (PT. DI) [2)]

Abstract

This paper presents the result of the normal modes analysis of N219 wing that has been done. It is done using computation method with stick model for wing structure. A short mathematical review of the normal mode analysis is presented in this paper. The analysis is done for two cases of flight, OEW and MTOW normal CG at maximum fuel. The results of analysis, natural frequencies and mode shapes, shown here, will be used for the dynamic analysis to know the flutter speed of this aircraft. The result shows that for the same frequency input of analysis, each two cases has different number of mode shapes, and different frequency in the same mode.
Key words : Normal Mode Analysis, Wing, Natural Frequency, Mode Shape, Computation Method, Stick Model.

1. INTRODUCTION

Normal mode analysis is a part of aeroelastic analysis which will used for the dynamic analysis [1]. Aeroelasticity is the study of static and dynamic behavior of structural elements in a flowing fluid [2]. Aeroelasticity in aerospace engineering is chiefly concerned with the interaction between the deformation of an elastic structure in an airstream and the resulting aerodynamic force [2]. Several years ago, Collar suggested that aeroelasticity could be use-fully visualized as forming a triangle of disciplines, dynamics, solid mechanics (elasticity) and (unsteady) aerodynamics. Aeroelasticity is concerned with those physical phenomena which involve significant mutual interaction among inertial, elastic and aero-dynamic forces. Other important technical fields can be identified by pairing the several points of the triangle. For example [3],

- Stability and control (flight mechanics) = dynamics + aerodynamics
- Structural vibrations (structural dynamic) = dynamics + solid mechanics
- Static aeroelasticity = steady flow aerodynamics + solid mechanics

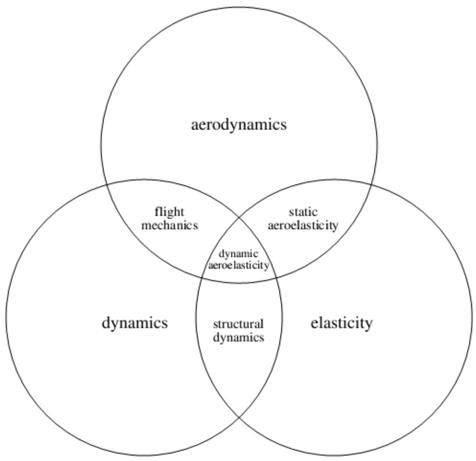


Figure 1-1 Schematic of the field of aeroelasticity [2]

During designing, manufacturing and examination of an aircraft prototype it is required to confirm that the aircraft is free from flutter in the range of the designed speed [1]. The flutter analysis and the flight test are used to confirm it [1]. Modeling phase is became a vital part to flutter analysis process due to the mode shapes and natural frequencies data that obtained from normal mode analysis in modeling phase [4]. The process for determine mode shapes and natural frequency can be done using free vibration equation which is contain stiffness and mass [4]. To find stiffness we could do a certain way, one of the way is find the cross-section inertia (Ix,Iy and J) and multiply with a certain modulus elasticity (E) [4]. The stiffness are use to be input for normal mode analysis before engineering do the flutter analysis [4].

The usual first step in performing a dynamic analysis is determining the natural frequencies and mode shapes of the structure with damping neglected. These results characterize the basic dynamic behavior of the structure and are an indication of how the structure will respond to dynamic loading [5].

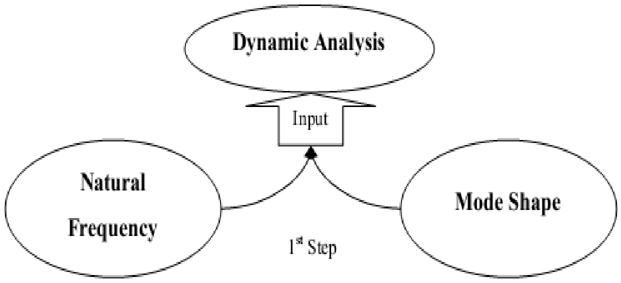


Figure 1-2 Input schematic of the dynamic analysis [6]

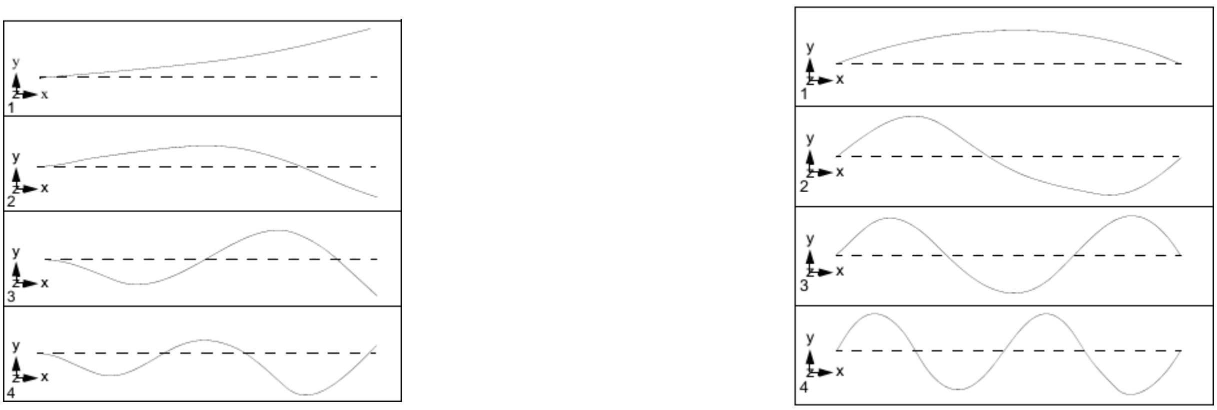
1.1. Natural Frequencies

The natural frequencies of a structure are the frequencies at which the structure naturally tends to vibrate if it is subjected to a disturbance. For example, the strings of a piano are each tuned to vibrate at a specific frequency. Some alternate terms for the natural frequency are characteristic frequency, fundamental frequency, resonance frequency, and normal frequency [5].

1.2. Mode Shape

The deformed shape of the structure at a specific natural frequency of vibration is termed its normal mode of vibration. Some other terms used to describe the normal mode are mode shape, characteristic shape, eigenvector and fundamental shape. Each mode shape is associated with a specific natural frequency [5].

Natural frequencies and mode shapes are functions of the structural properties and boundary conditions. A cantilever beam has a set of natural frequencies and associated mode shapes (Figure 1-2). If the structural properties change, the natural frequencies change, but the mode shapes may not necessarily change. For example, if the elastic modulus of the cantilever beam is changed, the natural frequencies change but the mode shapes remain the same. If the boundary conditions change, then the natural frequencies and mode shapes both change. For example, if the cantilever beam is changed so that it is pinned at both ends, the natural frequencies and mode shapes change (see Figure 1-3) [5].



(a) (b)

Figure 3. (a) The First Four Mode Shapes of a Cantilever Beam [5]
(b) The First Four Mode Shapes of a Simply Supported Beam [5]

This paper use normal mode analysis to determine the vibration characteristics (natural frequencies and mode shapes) of N219 wing structure B-11 configuration. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions [8]. There are many reasons to compute the natural frequencies and mode shapes of a structure. One reason is to assess the dynamic interaction between a component and its supporting structure. For example, if a rotating machine, such as an air conditioner fan, is to be installed on the roof of a building, it is necessary to determine if the operating frequency of the rotating fan is close to one of the natural frequencies of the building. If the frequencies are close, the operation of the fan may lead to structural damage or failure .

2. ANALYSIS METHOD

Procedure analysis of wing normal mode is shown as follow.

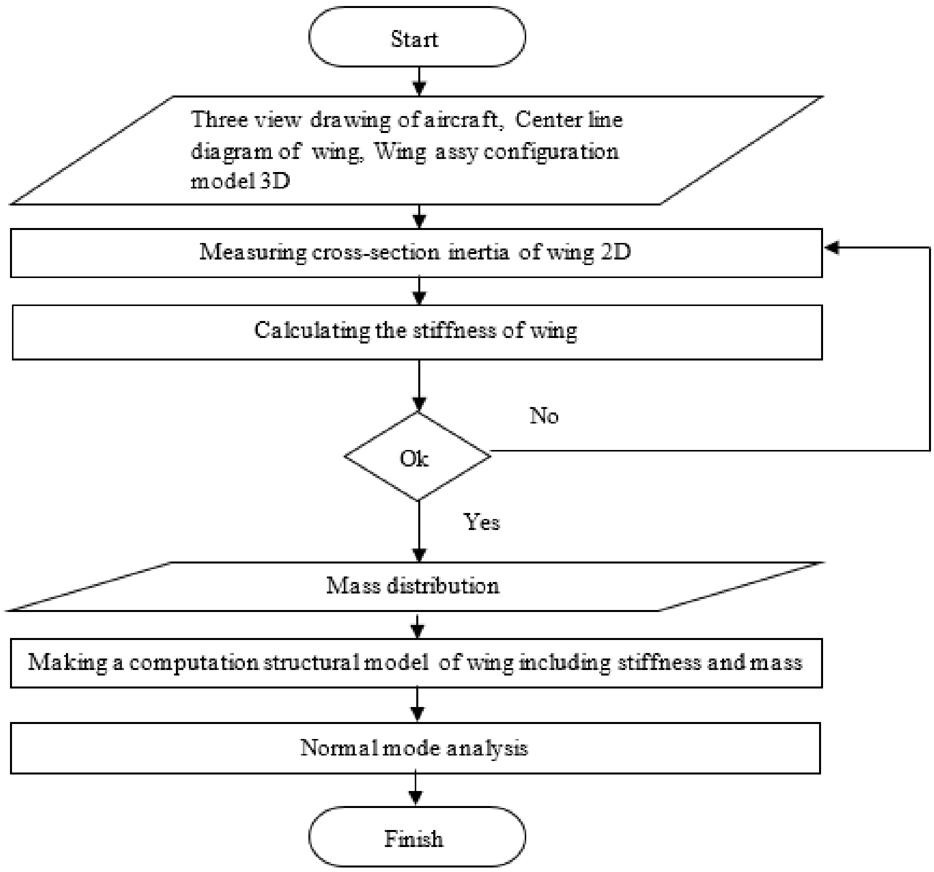


Figure 2-1 Procedure Analysis

2.1. Cross-section inertia

Cross-section inertia can be measured from 3D configuration model of wing by simplify the model which consist of skin, spar, ribs, and stringer. It is determined from 2D cross-section using computational program then we get Ixx, Iyy, J, and centroid values.

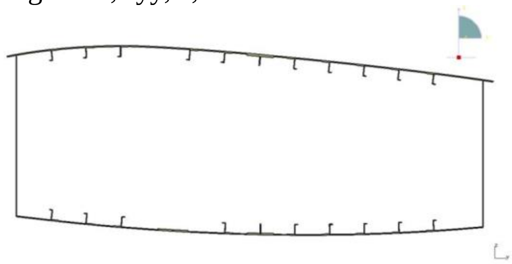


Figure 2-2 Cross-section of wing

2.2. Wing Stiffness (Rigidity)

For isotropic beams, calculation of the bending rigidity is a straight forward integration over the cross section, given by [7]

$$EI = \iint_A Ey^2 \, dA \tag{1}$$

where E is the Young's modulus. When the beam is homogeneous the Young's modulus may be moved outside the integration so that

$$\overline{EI} = EI \tag{2}$$

where I is the cross-sectional area moment of inertia about the z axis for a particular cross section. Here, the origin of the y and z axes is at the sectional centroid [7].

The torsional rigidity, denoted by GJ, is taken as given and may vary with x. For homogeneous and isotropic beams,

$$\overline{GJ} = GJ \tag{3}$$

where G denotes the shear modulus and J is a constant that depends only on the geometry of the cross section. To be uncoupled from bending and other types of deformation, the x axis must be along the elastic axis and also must coincide with the locus of cross-sectional mass centroids. For isotropic beams, the elastic axis is along the locus of cross-sectional shear centers [7].

Fig. 2-3 shows bending and torsional stiffness of the wing in certain distance. Horizontal axis shows the distance of cross-section from the root.

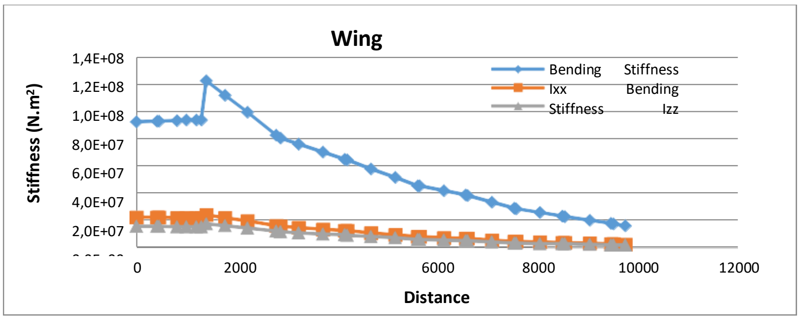


Figure 2-3 Stiffness of wing

2.3. Mathematical Overview of Normal Modes Analysis

The system of differential equation governing the elastic deformation of an airplane can be written as

$$[M]\ddot{u}(t) + [C]\dot{u}(t) + [K]u(t) = F(t) \tag{4}$$

where [M] is the mass matrix, [C] is the damping matrix, [K] is stiffness matrix, F(t) is the vector of external forces on the structure generated by the gust field, $\ddot{u}\ (t)$ is the vector of acceleration, $\dot{u}\ (t)$ is the vector of velocity, and u(t) is the vector of elasticdisplacements on the structure [12]. Eq. (3) for unforced motion is given by [7]

$$[M]\ddot{u}(t) + [C]\,(t) + [K]u(t) = 0 \tag{5}$$

The solution of the equation of motion for natural frequencies and normal modes requires a special reduced form of the equation of motion [5]. If there is no damping and no applied loading, the equation of motion in matrix form reduces to

$$[M]\{\ddot{u}\} + [u] = 0 \tag{6}$$

where [M] is the mass matrix, [K] is stiffness matrix, $\{\ddot{u}\ \}$ is the acceleration, and $[u]$ is the displacement [5]. This is the equation of motion for undamped free vibration . To solve Eq. (5) assume a harmonic solution of the form

$$\{u\} = \{\phi\} in\omega t \tag{7}$$

where $\{\phi\}$ is the eigenvector or mode shape and ω is the circular natural frequency [5]. If differentiation of the assumed harmonic solutionis performed and substituted into the equation of motion, the following is obtained [5]:

$$-\omega^2[M]\{\phi\}\sin\omega t + [K]\{\phi\}\sin\omega t = 0 \quad (8)$$

which after simplifying becomes

$$([K] - \omega^2[M])\{\phi\} = 0 \quad (9)$$

This equation is called the eigenequation, which isa set of homogeneous algebraic equations for the components of the eigenvector and forms the basis for the eigenvalue problem [5]. An eigenvalue problem
is a specific equation form that has many applications in linear matrix algebra. The basic form of an eigenvalue problem is

$$[A - \lambda I]x = 0 \quad (10)$$

where A is the square matrix, λ is the eigenvalues, I is the identity matrix, and x is the eigenvector [5]. In structural analysis, the representations of stiffness and mass in the eigenequation result in the physical representations of natural frequencies and mode shapes. Therefore, the eigenequation is written in terms of K, ω, and M as shown in Eq. (8) with $\omega^2 = \lambda$ [5].

There are two possible solution forms for Eq. (8) [5]:

a. If det $([K] - \omega^2[M]) \neq 0$, the only possible solution is
   $\{\phi\} = 0$ (11)
   This is the trivial solution, which does not provide any valuable information from a physical point of view, since it represents the case of no motion. ("det" denotes the determinant of a matrix.)
b. If det $([K] - \omega^2[M]) = 0$, then a non-trivial solution ($\{\phi\} \neq 0$) is obtained for
   $([K] - \omega^2[M])\{\phi\} = 0$
   From a structural engineering point of view, the general mathematical eigenvalue problem reduces to one of solving the equation of the form
   det $([K] - \omega^2[M]) = 0$ (12)
   atau
   det $([K] - \lambda[M]) = 0$ (13)
   dimana $\lambda = \omega^2$

The determinant is zero only at a set of discrete eigenvalues λ_i or ω_i^2 . There is an eigenvector $\{\phi_i\}$ which satisfies and corresponds to each eigenvalue. Therefore, can be rewritten as[5)]

$$([K] - \omega_i^2[M])\{\phi_i\} = 0 \qquad i = 1, 2, 3, \ldots \quad (14)$$

Each eigenvalue and eigenvector define a free vibration mode of the structure. The i-th eigenvalue λ_i is related to the i-th natural frequency as follows:

$$f_i = \frac{\omega_i}{2\pi} \quad (15)$$

where f_i is the i-th natural frequency and ω_i is $\sqrt{\lambda_i}$ [5)]. For undamped natural fequency

$$\omega = \sqrt{k/m} \quad \text{Eq. (16)}$$

where m is the mass, and k is the stiffness [11)]. The number of possible eigenvalues and eigenvectors is equal to the number of degrees-of-freedom that have mass or the number of dynamic degrees-of-freedom [5].

2.4. Structural Model of Wing

Structural model in normal modes analysis can be done with several methods, which generally used are full FEM, hybrid, and stick model [6]. In this paper, structural model using stick model method.

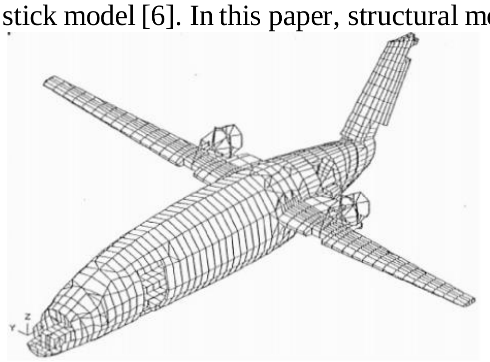


Figure 2-4 Structural model of aircraft with full FEM model [6]

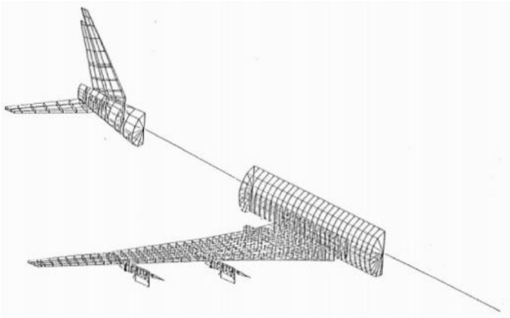


Figure 2-5 Structural model of aircraft with hybrid model [6]

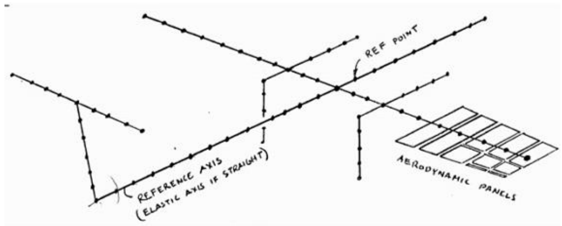


Figure 2-6 Structural model of aircraft with stick model [6]

An airplane with slender parts with rigid cross-section could be modeled by a number of beams placed along reference axis of the structure. Reference axis is elastic axis of the structure. Elastic axis is a number of shear center position of cross section structure. In other word, stick model is simplify of full FEM structural model, so its accuracy of analysis result doesn't equal to full FEM. Simplifying model is needed in design process, because a more simple model, a less time needed to analyze.
Structural model of wing including stiffness, shell, and mass can be seen in fig. 2-7 and fig. 2-8 below.

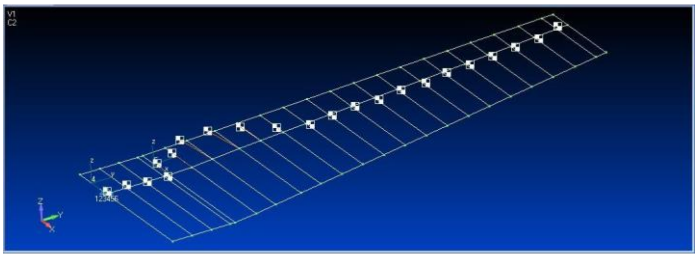


Figure 2-7 Structural model of aircraft wing

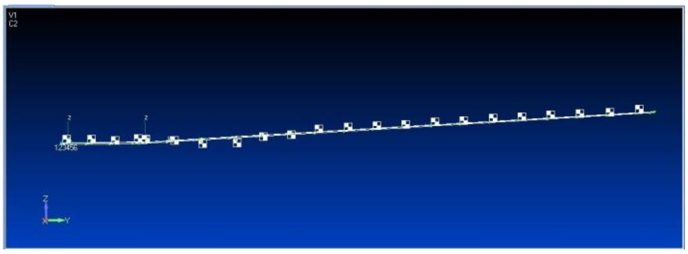


Figure 2-8 Back view of wing model

2.5. Normal Mode Analysis

The mathematical overview for normal modes analysis has been explained in part 2.3. This paper used MSC NASTRAN for computational normal modes analysis. It has been done for two cases of flight, OEW (Operating Empty Weight) and MTOW (Maximum Take Off Weight) normal CG at maximum fuel. The frequency input for this analysis is stated of 50 Hz. Structural model of wing for OEW can be seen at fig. 2-7, for MTOW normal CG at maximum fuel can be seen at fig. 2-9 below.

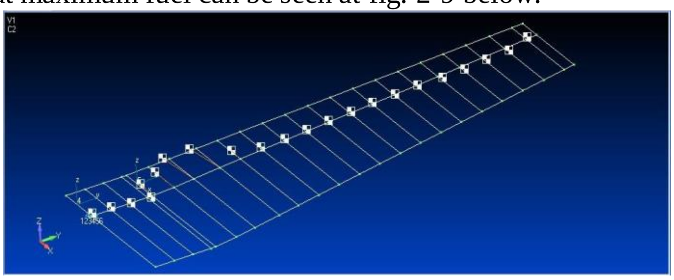
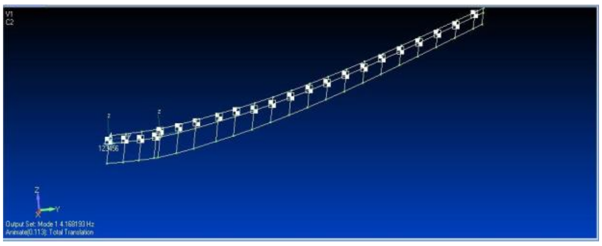


Figure 2-9 Structural model of wing for MTOW normal CG at maximum fuel

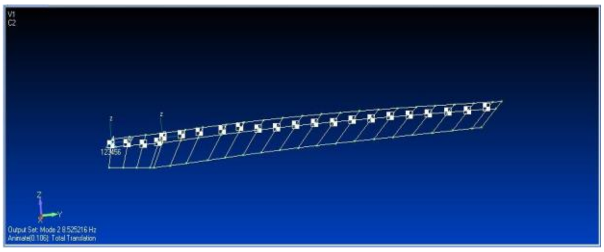
3. RESULT AND DISCUSSION

3.1. Mode Shapes

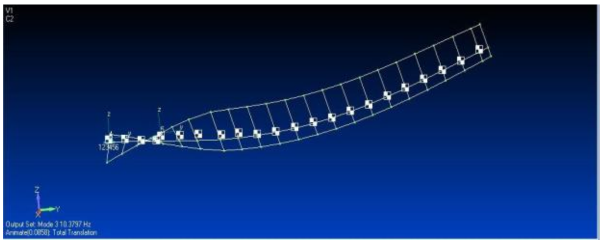
The first eight mode shapes of wing at OEW can be seen at fig. 3-1 below.



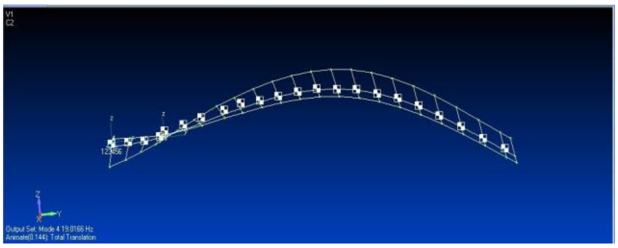
Mode 1



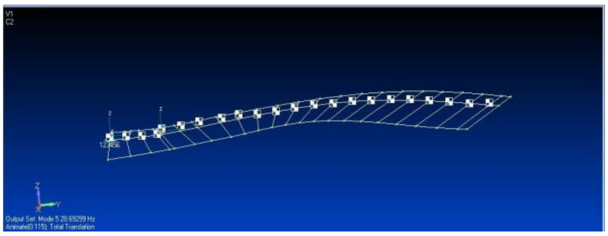
Mode 2



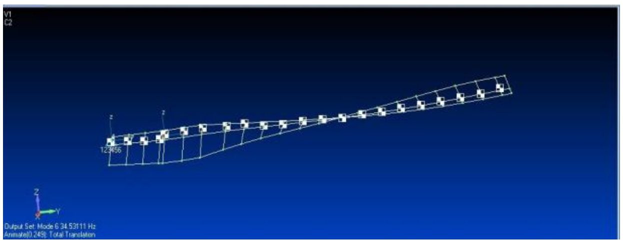
Mode 3



Mode 4

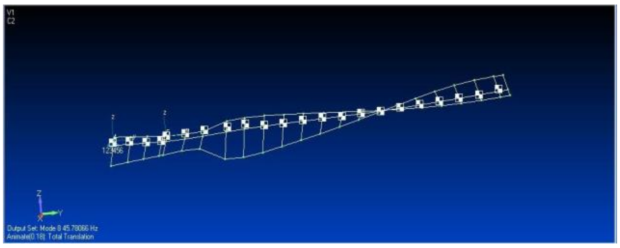


Mode 5



Mode 6

Mode 7



Mode 8

Figure 3-1 Mode shape of wing at OEW case

The mode shapes of wing from fig. 3-1 are :

a) Vertical bending (Mode 1)
b) Inplane bending (Mode 2)
c) Torsion (Mode 3)
d) Second vertical bending (Mode 4)
e) Second inplane bending (Mode 5)
f) Second torsion (Mode 6)
g) Third vertical bending (Mode 7)
h) Third torsion (Mode 8)

The first nine mode shapes of wing at MTOW normal CG at maximum fuel can be seen at fig. 3-2 below.

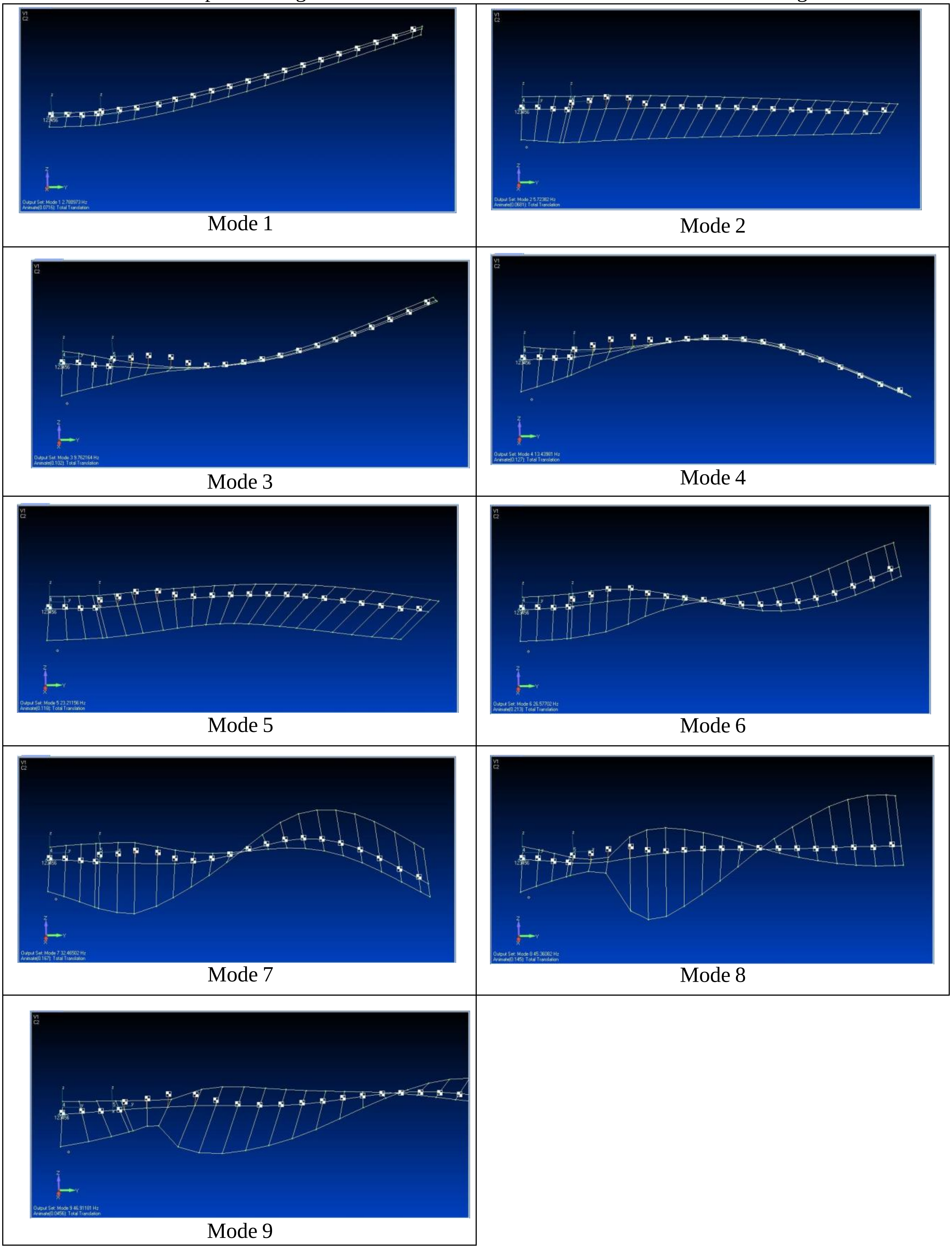


Figure 3-2 Mode shape of wing at MTOW normal CG at maximum fuel case

The mode shapes of wing from fig. 3-2 are :

a) Vertical bending (mode 1)

b) Inplane bending (mode 2)
c) Torsion (mode 3)
d) Second vertical bending (mode 4)
e) Second inplane bending (mode 5)
f) Third vertical bending (mode 6)
g) Vertical bending (mode 7)
h) Second torsion (mode 8)
i) Third inplane bending (mode 9)

3.2. Natural Frequency

Each case of flight have several mode shapes with different natural frequencies. Table 3-1 shows the natural frequencies of wing at OEW and MTOW at maximum fuel cases which have been analyzed.

Table 3-1 Natural Frequencies at each Mode Shape of Wing

MODE	Natural Frequency	
	OEW	MTOW-N CG AT MF
1	4.168	2.781
2	8.525	5.724
3	10.380	9.762
4	19.017	13.440
5	28.693	23.212
6	34.531	26.577
7	42.808	32.465
8	45.781	45.361
9		46.912

Table 3-1 shows that the natural frequency at the same mode shape different for each case. Greater mass have lower frequency at the same mode and vice versa. It is appropriate to the equation of natural frequency at Eq. (16). It shows that natural frequency is equal to stiffness divided by mass, it means that structure with the same stiffness would has lower frequency if the mass greater.

4. CONCLUSION

Normal modes analysis of wing in this paper shows that aircraft wing have eight mode shape with different natural frequency for each mode at OEW case and nine mode shape with different natural frequency for each mode at MTOW at maximum fuel. The results of the normal mode analysis for the N219 wing will be used for the flutter analysis.

ACKNOWLEDGEMENT

The authors would like to thank to Mr. Gunawan as the director of Aeronautics Technology Center, Mr. Agus Nurtjahjomuljo as the director of Aerostructure Department, and Mr. Agus Aribowo as the Head of N219 aircraft program. Special thanks to aeroelasticity engineers of IAe (PT. Dirgantara Indonesia) and Mr. M. Sjamsoel Arifin as the supervisor of Aeroelasticity Department of IAe.

REFERENCES

1) Dimitrijević, J. and Kovačević, P.: *Computational Modal Analysis of the LASTA Aircraft*, Scientific Technical Review, Vol.60, No.1, pp.60-69, 2010.
2) Dewey H., G. Alvin Pierce, Olivier A., and Marilyn J.: *Aeroelasticity Class Notes*, Atlanta, Georgia, 2006.
3) Clark, R., Cox, D., et_al.: *A Modern Course in Aeroelasticity*, Kluwer Academic Publisher, New York, 2004.

4) Patriot, W..: *Nxxx-Bxx Preliminary Vertical Tail Stiffness Analysis For Dynamic Model Analysis* – 1st Iteration, IAe, Bandung, 2014.
5) MSC Software*: MSC Nastran 2012 Dynamic Analysis User's Guide*, MSC Software Corporation, USA, 2011.
6) Sakti, B.: *Modul Training Structural Dynamic Analysis*, IAe, Bandung, 2013.
7) Dewey, H., Alvin, P.: *Introduction to Structural Dynamics and Aeroelasticity*, Cambridge University Press, New York, 2011.
8) Prabhushetty, V., and Mangshetty, S.: *Static and Normal Mode Analysis of Aircraft Avionics Compartment Door*, International Journal of Engineering Science Invention, Volume 3 Issue 8, PP.37-43, 2014.
9) Banerjee, J.R., and Butler, R.: *Flutter Analysis Of Composite Wings Using Symbolic Computation*, ICAS CONGRESS, 2000.
10) Wright, R., and Cooper E.: *Introduction to Aircraft Aeroelasticity and Loads*, Jhon Wiley & Sons Ltd, England, 2007.
11) Guillaume, P.: *Modal Analysis*, Vrije Universiteit Brussel, Brussel, Belgium.
12) Rodrigues, E.A., and Kamiyama, M.T.: *Computation of Dynamic Loads on Aircraft Structure due to Continous Gust Using MSC/NASTRAN*, The MacNeal-Schwendler Corporation, Brazil, 1997.
13) Qin, Z.: *Vibration and Aeroelasticity of Advanced Aircraft Wings Modeled as Thin-Walled Beams– Dynamics, Stability and Control*, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 2001.